45-988 Competitive Strategy Project

Carnegie Mellon University - GSIA

Prof. Jeffrey Williams

Fall 2002

Game Theory for Managers

Some review, applications and limitations of Game Theory

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12/20/02 – Version 1

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Introduction

This paper was written for the class 45-988 Competitive Strategy Project with Prof. Jeffrey Williams. Its goal is to present Game Theory to a manager with limited knowledge in this subject. Here I present a basic review of the theory, demonstrate some examples and discuss some difficulties that managers can have and how to address them.

Game Theory is a debatable theory in terms of real applicability, mainly because of its strong assumptions and lack of real published cases. However, it provides important insights for companies to compete more effectively and for managers to think more strategically. In an easy language, my goal with this paper is to show those advantages for managers[1].

Section 1 presents a summary of definitions and assumptions of Game Theory.

Section 2 shows how a game is structured and provides examples of different kinds of games and solutions. The objective of this section is not to cover all game-theoretic analysis, but to create a common language and understanding to the following sections. Most of the examples are simplified just to teach the reasoning of Game Theory.

Section 3 presents some benefits of Game Theory.

Section 4 discusses important issues about applications and limitations that a manager and a student must know. This is the main part of this paper, but also does not solve all problems of Game Theory. The objective of this section is to motivate the discussion and to provide some advices for a basic reader about how to face some limitations.

Section 5 concludes with some check lists for practical use.

Appendix A presents a Chronology of Game Theory. Basically, it is a replication of my source and it is here just to provide the reader with more sources, besides my reference section.

Appendix B shows the site ComLabGames, a useful tool for professors and students, even managers, to design and test game-theoretic situations.

[1] As Game Theory has many sources, I will continue with my research to complement this work and to write other versions.

1. Game Theory[2] – definitions and assumptions

In the business world, companies make decisions and actions to maximize their payoffs, often represented by profits, revenues, market share, market value, etc. In most competitive situations, companies' payoffs depend not only on their own actins, but also on the actions of other competitors that are pursuing their own objectives.

"Game theory is the study of interactions among players whose payoffs depend on one another's choice and who take that interdependence into account when trying to maximize their respective payoffs" (Ghemawat, 1999)

It seems obvious to analyze the consequences of our own actions and competitors' reactions. An intuitive way to do this analysis is to use some information about competitors, anticipate their likely actions (or reactions) and choose the best moves.

However, it is not an easy task – managers need data for payoff calculations, experience about competitors' behavior and some methodology. It is in the methodology part where Game Theory helps managers structure their reasoning. This theory serves as a framework for competitive analysis that takes into account the interaction of strategies.

"Game Theory offers a scientific approach to strategic decision-making. In place of the anecdotes, cases, stories, and examples that are commonly offered as advice to negotiators, Game Theory gives a systematically structure view" (McMillan, 1992).

It imposes an assumption typically made in economics: each player acts to maximize his payoff. Moreover, as companies' payoffs depend on the actions of the other companies, Game Theory assumes the each player reasons to figure out all competitors' options and to anticipate what they are going to do.

To do so, you need to put in your rival's shoes and reason what he would do to maximize his payoffs. So, there is another assumption: you should know your competitors' motivation, payoffs and capabilities. In fact, it assumes that your competitors knows yours either.

[2] Appendix A shows a chronology of Game Theory related to academic papers

In summary, Game Theory assumes that:

- Each player has clear preferences and acts to maximize his utility (utility maximization and rationality)
- Each player knows the other's utilities and preferences (common knowledge)

In short, all strategic players are assumed to be rational and self -interested. This means that players want to get as much utilities for themselves as possible, and are quite clever in figuring out how best to accomplish this objective.

Game Theory is a good tool for understanding how decisions affect each other. Dixit and Skeath (1999) stated the three uses of Game Theory:

- "Explanation – when the situation involves interaction of decision makers with different aims, Game Theory supplies the key to understanding the situation and explains why it happened.
- Prediction – when looking ahead to situations where multiple decisions makers will interact strategically, people can use Game Theory to foresee what actions they will take and what outcomes will result.
- Advice or Prescription – Game Theory can help one participant in the future interaction, and tell him which strategies are likely to yield good results and which ones are liable to lead to disaster."

Next section presents some examples of games and solutions following Game Theory's assumptions.

2. Games –Structure and Examples

Many books and articles start explaining Game Theory with analogies with sports, games, cards, movies, etc. I acknowledge that these analogies are helpful to create a general understanding, to provide some insights and to think strategically in multiple environments in life. However, I decided to limit all examples in this paper to company's situations because I want the reader to focus on his business set, generating insights for his practical use.

First, it is important to establish a common language and general understanding of a game. Some names the reader should have in mind (LeClair, 1995):

- Player — A rational and self interested decision-making entity.
- Strategy — A rule that tells the player which action to choose at each instant of the game.
- Outcome — The result of every possible sequence of actions by the players.
- Payoff — The personal satisfaction obtained from a particular outcome.
- Equilibrium — The "best" combination of choices.

As you will see, a formal game theory model consists of the following (Rasmusen, 1989 in LeClair, 1995):

- a set of players, a set of actions and strategies for each player,
- a description of the order of play and
- the information available to any player at any point in the game,
- the outcome that results from every possible sequence of actions by players,
- a ranking of each outcome to every player (the player's payoffs), and
- a solution (or equilibrium) concept

All those definitions will be better understood with the next examples. The objective of this section is to explain the mechanics and reasoning of types of interactions. All examples are very simplified, not necessary based on past real cases, and the payoffs are given. Also, there are not complex math in the solutions. Since the goal of this paper is to discuss some issues about benefits, applications and limitations in order to give some advices to managers, I didn't include all elements and examples as in a formal game theory course.

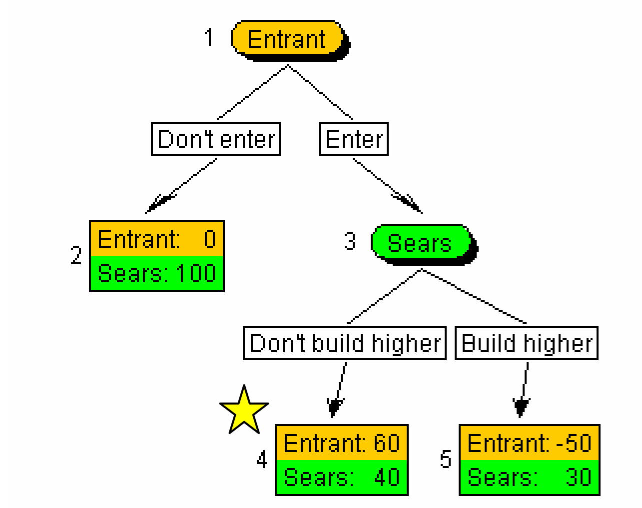
If you do not know much about Game Theory, this section provides you with a good overview, so that you can deep your knowledge in specific books. People who know the theory can skip this section.

2.1. Example 1 – Sequential game - Strategic Entry Deterrence[3]

Game theory distinguishes two major categories of games: simultaneous games, where all participants make their decisions simultaneously, or sequential games, where participants react to each other's actions in turn.

The Sears Tower in Chicago is currently the tallest building in the United States. This status endows the building with a special form of prestige, enabling its owners to command higher rents than in otherwise similar office buildings. Suppose that an Entrant is considering whether to build an even taller building. Suppose also it knows that any firm that has permanent ownership of the tallest building will earn a large economic profit. Its concern, naturally, it that Sears (or some other firm) may build a still taller building, which would substantially diminish the Entrant's payoff.

It is a sequential game because Entrant must choose first, and Sears will know the Entrant's choice to make its decision. The game can be modeled in a game-tree shown in picture 1, called extensive form[4], that shows all possible options and outcomes of each option. There is a label with numbers in each element, called node.



Picture 1

You can see that the Entrant (node 1) must decide between Enter and Don't Enter in this market, i.e., to build a taller tower or not. If it chooses Don't Enter, the game ends in the node 2. If its chooses Enter, then Sears (node 3) has two options, Don't build it (node 4) or Build a higher building (node 5).

3 Example from Frank, 2000, with some modifications to fit some explanations

4 All schematic examples were drawn in the software from the site ComLabGames (http://www.cmu.edu/comlabgames/), an open tool created by Miller and Prasnikar from Carnegie Mellon University, Graduate School of Industrial Administration. The software is for instructors and students to design, run, and analyze the outcomes of games played over the Internet. See more details in Appendix XX

Payoffs are necessary for players to make their decisions. Since game theory involves formal reasoning, we must have a device for thinking of utility maximization in mathematical terms.

If Entrant does not enter, nothing changes in the current situation – Sears gets a payoff of 100, and Entrant gets zero (node 2). If the Entrant enters and Sears don't compete by building an even higher tower, then Entrant has advantage and captures a payoff of 60, while Sears gets 40 (node 4). If Sears build a higher one, then Entrant loses money with a payoff of -50 and Sears get 30 (node 5).

Sears naturally wants Entrant not to enter because it prefers the payoff of 100 (node 2), but this decision depends only on the Entrant. How does the Entrant must decide? It must use the concept of Backward Induction, that is looking ahead and reason back.

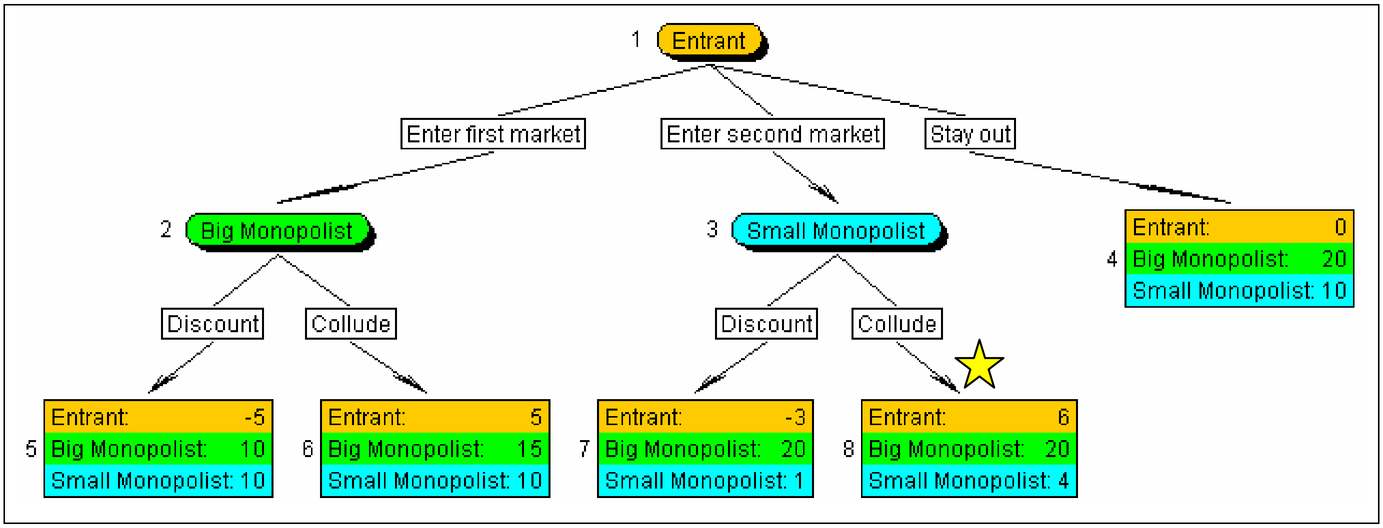
For example, looking to the Sears' choices, and assuming that Sears wants to maximize its payoff, Sears will prefer to not build a higher building because the payoff of 40 (don't build) is greater then 30 (build). Entrant knows that Sears will think in this way, so the Entrant's payoff will be 60 (node 4). Then, Entrant knows that, if it chooses Don't Enter, it will get zero, if chooses Enter, it will get 60. As a result, it will prefer Enter and build a tower. The equilibrium of this game is in node 4.

Note that there are some simplifications, since there are many alternatives in real life. For example, the Entrant could build a small building, or Sears could build another tower even if Entrant does not enter, or it could build a small one if Entrant enters. However, those options do not capture this competitive situation and they are irrelevant in this analysis.

2.2. Example 2 – Sequential game - Strategic Entry Deterrence with 3 players[5]

In this example, there are three players, an Entrant and two retailers, respectively called Big Monopolist and Small Monopolist, which currently hold regional monopolies in the localities they serve. The schematic representation in a game-tree is shown in picture 2.

[5] Example from Miller and Prasnikar, 2004, with some modifications to fit some explanations



Picture 2

The Entrant can decide to enter or not in the market. If it chooses not to enter, the absence of competition provides the Big and Small Monopolist with payoffs of $20 and $10 million, respectively, in present value. In this case, payoff of Entrant is zero (node 4).

Now the reader knows how to interpret all options and respective payoffs by looking the game tree, so we will simplify and do not explain all business situations anymore. The objective here is to explain the mechanics of games and then next chapter discuss some important issues.

To solve the problem, if you are the Entrant, the reasoning is the same – the backward induction. Check game tree and payoffs to verify that:

- If Entrant chooses Enter First Market, Big Monopolist will choose Collude to get payoff of 15
- If Entrant chooses Enter Second Market, Small Monopolist will choose Collude to get payoff of 4

So, you predict Sears' choices and you know that you have 3 possible rational outcomes:

- Zero if Stay Out (node 4)
- 5 if Enter First Market (node 5)
- 6 if Enter Second Market (node 8)

Therefore, the strategy that maximizes the Entrant's payoffs is to enter in the second market because the Entrant knows that the Small Monopolist will choose to collude.

Notice that, in this case, Big Monopolist does not play the game. It shows that not always all players act in the decision, but we need to consider them in the game tree to predict the best choices.

The reasoning to solve more complex sequential game trees is the same. You start from the bottom and go up doing simplification in the tree since you know the likely choices of all players using the assumptions of utility maximization, rationality and common knowledge.

Example 3 – Simultaneous Game – Prisoner's Dilemma

One of the most frequently discussed examples of pursuing self-interest is the so-called prisoner's dilemma. The mathematician A.W.Tucker is credit with having discovered this simple game, whose name derives from the anecdote originally used to illustrate it (Frank, 2000). Two prisoners are held in separate cells for a serious crime that they did, in fact, commit. The prosecutor, however, has only enough hard evidence to convict them of a minor offence, for which the penalty is a year in jail. Each prisoner is told that if one confesses while the other remains silent, the confessor will go free while the other will spend 20 years in prison. If both confess, they will get an intermediate sentence of 5 years.

As the two prisoners are not allowed to communicate with one another, they need to decide simultaneously without knowing what the other is thinking. In a simultaneous game, one must try to predict what are the action the other players are going to make, realizing that they are trying to predict what one is going to take.

In this simultaneous game, a better representation of it is a payoff matrix in picture 3. This representation is called strategic form.

		Prisoner A	
		Remain silent	Confess
Prisoner B	Remain silent	1 1	0 20
	Confess	20 0	5 5

Picture 3

Since it is not a sequential game, the backward induction method does not apply. To solve, each Prisoner need to think what the other is thinking to make a decision.

However, there is an easy solution. Suppose you are Prisoner A.

- If Prisoner B chooses Remain Silent, your best choice is Confess (zero is better then 1 year in jail)[6]
- If Prisoner B chooses Confess, your best choice is Confess (5 is better then 20 years in jail).

Notice that Confess is the best alternative regardless what Prisoner B chooses. So, Confess is called Dominant Strategy, which means that a strategy that produces better results no matter what strategy the opposing player follows.

Meanwhile, Prisoner B is thinking in the same way, and Confess for him is the best choice. Therefore, the equilibrium will be to confess for both, i.e., they will get 5 years in prison each. You noticed that this is not the best that they could get if they communicate and remain in silence, getting just one year in prison. When each behaves in a self-interest way, both do worse then if each had shown restraint. However, would you take the risk and choose to remain silent?

2.4. Example 4 – Simultaneous Game – Prisoner's Dilemma in two business game

Game with Prisoner's Dilemma can be found in business situations, such as pricing setting or in advertising decision. See the two examples as following.

Consider two firms, Coca-Cola and Pepsi[7], selling similar products. Each must decide on a pricing strategy. They best exploit their joint market power when both charge a high price; each makes a profit of $10 million per month. If one sets a competitive low price, it wins a lot of customers away from the rival. Suppose its profit rises to $12 million, and that of the rival falls to $7 million. If both set low prices, the profit of each is $9 million. The payoffs are represented in the picture 4.

[6] Obs: the players here are trying to minimize their payoffs. In this particular case, 1 is better then 20. Another representation would be writing all payoffs in negative numbers (after all, it represent years in jail). Then, -1 is greater then -20.
[7] Example found in http://www.econlib.org/library/Enc/PrisonersDilemma.html, by Avinash Dixit and Barry Nalebuff , with some modifications to fit the explanations.

		Coke	
		High price	Low price
Pepsi	High Price	10 10	12 7
	Low Price	7 12	9 9

Picture 4

Here, the low-price strategy is similar to the prisoner's confession, and the high-price is similar to keeping silent. Then low-price is each firm's dominant strategy, but the result when both set low-price is worse for each than that of both cooperating in high-price.

Consider this second example[8]. When a firm advertise its products, its demand increases for two reasons. First, people who never used the product start buying. Second, people who already consume it switch brands. That is why advertising is important and crucial for companies to compete, but in the same time it can be too costly and profits depends on the right combinations of advertise cost and revenues that is generated by this expense.

The picture 5 shows the profits to hypothetical companies under four possible combinations of their advertise / don't advertise decisions. If both firms advertise, each earns a profit of only 250, as compared with a profit of 500 each if neither advertise. If just one of the firms advertises, it earns higher profits (750) because it captures customers from the competitor.

		Firm 1	
		Advertise	Don't Advertise
Firm 2	Advertise	250 250	0 750
	Don't Advertise	750 0	500 500

Picture 5

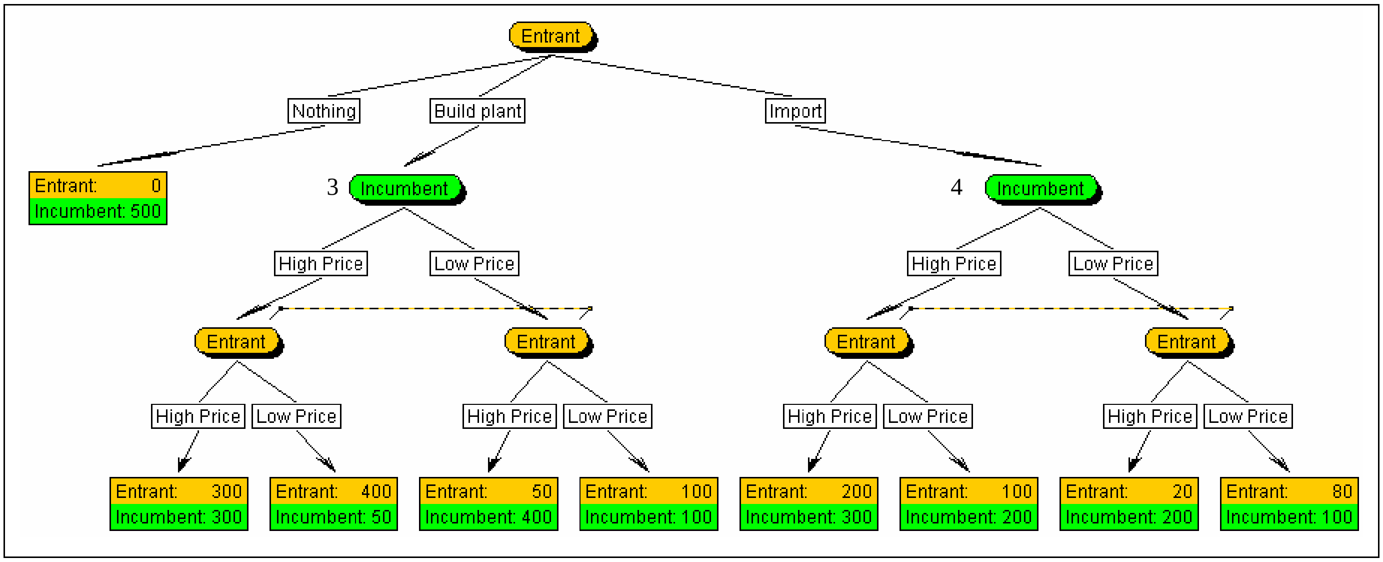
[8] Example from Frank, 2000, with some modifications to fit some explanations

Due to the incentives confronting the individual firm, the dominant strategy is Advertise for both. It is a typical Prisoner's Dilemma, where the payoffs are not greater than if they don't advertise[9].

2.5. Example 5 – Combining Sequential and Simultaneous Game and Dominant Strategy

A game is not necessary only sequential or simultaneous; it can have part of both. Consider the game of picture 6. Entrant 1 considers if it enters in the market where the Incumbent dominates. Today Incumbent's payoff is 500 and Entrant's is zero. To enter, Entrant has two options: build a plant to produce the competitive product domestically or import it.

So far, it is a sequential move because the Incumbent will know the Entrant's actions to build a plan or import. However, after constructing the plan or set the importation process, they need to set a price and this decision is taken simultaneously.



Picture 6

The simultaneous action is characterized by the dotted lines in Entrant's nodes. To solve this game, the best way is to analyze each sub-game. First, we solve the simultaneous game below the node 3 by looking in its strategic form in picture 7. Check and compare payoffs and nodes to see that it is possible to transfer from an extensive form to a strategic form. In fact, the strategic form is better to visualize the dynamic and sequence of interaction; the strategic form is better to easily find the equilibrium.

9 The prisoner's dilemma is an example of a broader class of problems called commitment problems (Frank, 2000). The common feature of these problems is that people can do better if they commit themselves to behave in a way that will later be inconsistent with their own material interests. One solution is to communicate and decide jointly for high prices. It would be a matter of trust and commitment. How would you know if your rival will not cheat? This paper does not cover this issue, but there are many books about this.

		Entrant	
		High price	Low price
Incumbent	High Price	300 300	400 50
	Low Price	50 400	100 100

Picture 7

The equilibrium here is Low Price, Low Price. It is a typical prisoner's dilemma. Low Price is the dominant strategy for both companies. Then, the Entrant gets payoff of 100.

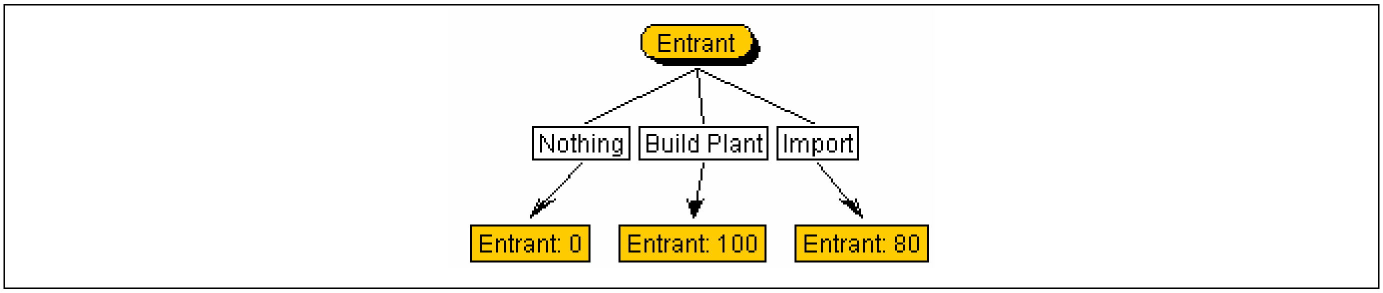
Next, we solve the other simultaneous game after the node 4. Look picture 8.

		Entrant	
		High price	Low price
Incumbent	High Price	200 300	100 50
	Low Price	20 400	80 100

Picture 8

In this case, Entrant has no dominant strategy. However, Incumbent still has a dominant strategy - Low Price. The Entrant knows that, so the best alternative since Incumbent will choose Low Price is also Low Price. The Entrant gets payoff of 80.

To finally decide, it is just to return to game tree and compare the three alternatives, as in picture 9.



Picture 9

If Entrant does nothing, gets zero. If it builds the plan, it gets 100. If it imports, it gets 80. Therefore, it best choice is to build the plan and enter in the market with low price.

So far, you should have understood the reasoning and mechanics to analyze different kind of games, our objective. From now on, next examples present different scenarios and they will be more simplified in terms of business explanations. It will be shown just the schematic representations to focus on the differences from those we have mentioned.

2.6. Example 6 – Iterative Dominance and Nash Equilibrium[10]

Consider two companies A and B that need to choose between produce low, medium and high quantities of a given product. Payoffs of all combined strategies are in Picture 10.

		Company A		
		Low Quantity	Medium Quantity	High Quantity
Company B	Low Quantity	18 19	20 15	17 9
	Medium Quantity	14 20	16 16	15 8
	High Quantity	9 18	7 12	0 0

Picture 10

First, there is not any dominant strategy for both players. None strategy present a best payoff regardless the others' move. However, look at the payoffs of Company B in High Quantity option: all numbers are lower then its own other strategies (low and medium). In this case, we say that High Quantity is a Dominated Strategy.

[10] Example from Miller and Prasnikar (2004) with some modifications in the payoff structure

Therefore, we can eliminate this option and we can redraw the game without it. See now Picture 11.

		Company A		
		Low Quantity	Medium Quantity	High Quantity
Company B	Low Quantity	18 19	20 15	17 9
	Medium Quantity	14 20	16 16	15 8

Picture 11

Company A still has 3 options, and Company B has 2. Again, we look for new Dominant and Dominated strategies. Now, Company A has a Dominant Strategy – Medium Quantity, it is a strategy that give higher payoffs regardless Company B's actions.

Company B knows that Company A will choose Medium Quantity. So, the best response to this option is to choose Medium Quantity. Both companies will get a payoff of 16. This method of eliminating options is called Iterative Dominance.

Consider now a new game of Picture 12. The payoffs of Company B High Quality are slightly different. Now, there is no Dominated Strategy for Company B anymore. In fact, there is no Dominated or Dominant strategy for any company. Even though, there is an optimum solution.

		Company A		
		Low Quantity	Medium Quantity	High Quantity
Company B	Low Quantity	18 19	20 15	17 9
	Medium Quantity	14 20	16 16	15 8
	High Quantity	9 21	7 17	0 0

Picture 12

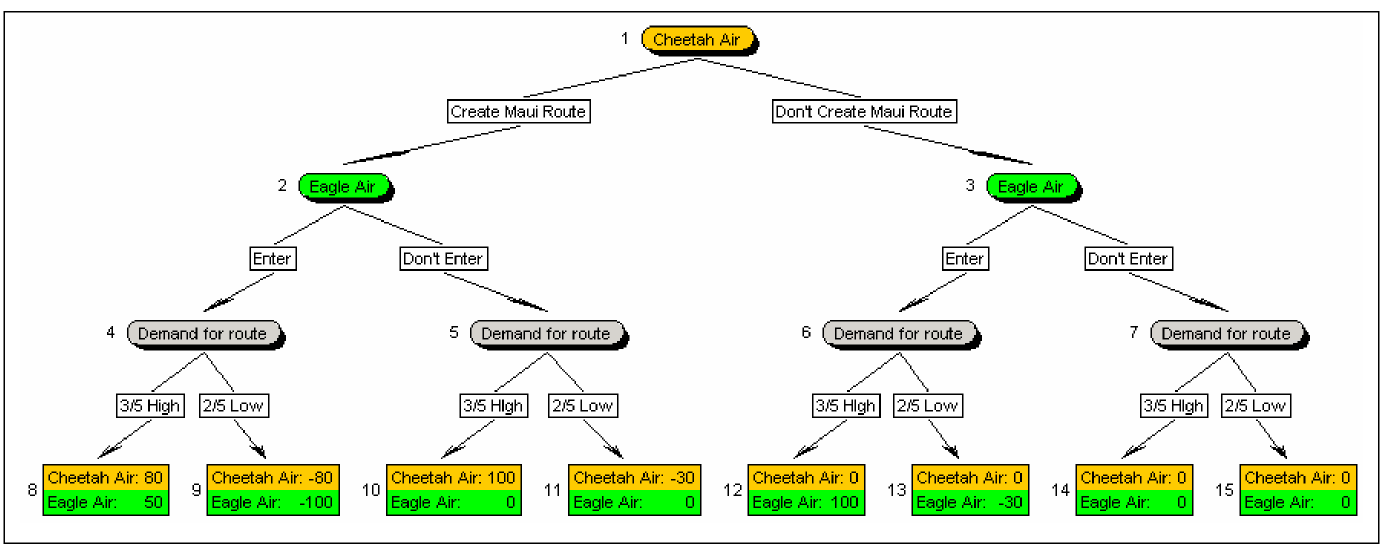
This concept is called Nash Equilibrium and is based on the concept that both players reason to find out what is the best response for each strategy.

For example, if Company B chooses Low Quality, the best response for Company A is Medium Quality. Knowing that, if A chooses Medium Quantity, so B will prefer High Quantity. Now, if B prefers High Quantity, the best response for A is to choose Low Quantity. To end the game, if A chooses Low Quantity, the best for B is remain in High Quantity. Therefore, Low Quantity for A, and High Quantity for B is the optimal solution, i.e., the Nash Equilibrium.

This equilibrium is achieved by beginning the reasoning from any point, from any company. Note that a change in some payoffs modifies the solution of the game. Also, you can ask: how company B is sure that A will choose Low Quantity in order to choose High Quantity. Well, there is no reason for A to act differently is A knows that B prefers High Quantity. It is the same reasoning for the opposite question. Here we assume that both players put in the shoes of the competitor to decide the best response. We will cover the issues of modifying payoffs and rationality later.

2.7. Example 7 – Uncertainty and Expected Payoffs[11]

Look this game at Picture 13. It is the competition about two airlines. Cheetah airlines is considering if it creates a new route to Maui or not. Eagle Air can wait and take its decision if it will enters. However, none of the companies knows if there would be high or low demand for Maui route. So, what is important is to quantify the probabilities.



Picture 13

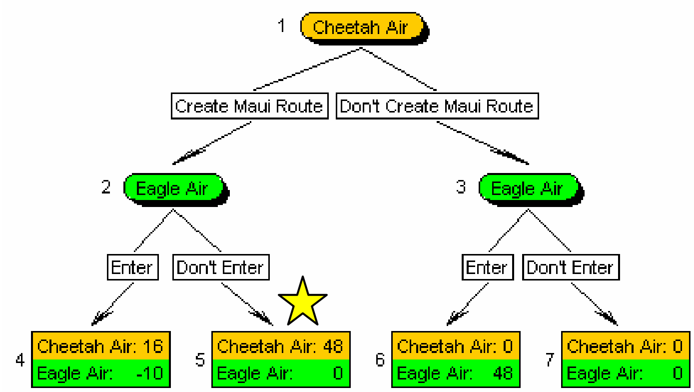
[11] Example from Miller and Prasnikar (2004) with some modifications in the payoff structure

There is 60% of probability that customers use the route (high demand) and 40% probability for low demand. The payoffs are stated in the extensive form of this game.

To solve, it is necessary to calculate the expected payoffs. For example, in node 4:

- Expected payoff for Cheetah Air is 16 (=3/5*80 + 2/5*-80)
- Expected payoff for Eagle Air is -10 (=3/5*50 + 2/5*-100)

Doing the calculations for all nodes, it is possible to simplify the game as presented in Picture 14. Notice that there is only four possible outcomes using expected payoffs.



Picture 14

Now, it is just to solve using backward induction. The equilibrium is node 5, with Cheetah Air creating Maui Route and Eagle Air not enter in this market.

2.8. Example 8 – Repeated Games and Mixed Strategies[12]

Consider two companies that need to decide for High or Low prices, as in Picture 15.

		Company A	
		High	Low
Company B	High	75 75	125 -50
	Low	75 50	25 25

Picture 15

[12] Example from Prasnikar (2002)

Notice that there is no dominant strategy and no Nash Equilibrium. What to do? If it is a repeated game, there is a easy solution. A repeated game is simply a game made up of a finite or indefinite repetition of a one-shot game. In this situation, a player's utility is optimized through use of a *mixed* strategy, in which he change options using a weighted strategy. Mixing is necessary whenever maximization of the player's utility depends on creating *uncertainty* in the expectations of her opponent.

Consider q the probability for Company A chooses High, and 1-q is the probability to choose Low. Similarly, p is the probability for Company B chooses High, and 1-p is the probability to choose Low.

	Company A	**q**	**1- q**	
	Company B	High	Low	**EU_B:**
p	High	75, 75	125, -50	**75*q – 50*(1-q)**
1- p	Low	75, 50	25, 25	**50*q + 25*(1-q)**
	EU_A:	**75*p + 75*(1-p)**	**125*p + 25*(1-p)**	

Picture 16

Here it is important the concept of Expected Utility. Company A will only mix between High and Low if EU(High) = EU(Low). It requires to Company B to mix so that:

- 75*p + 75*(1-p) = 125*p + 25*(1-p) ➔ p = ½
- The expected payoff for Company A is 75 if it mixed with 50% between High and Low.

Company B will only mix between High and Low if EU(High) = EU(Low). It requires to Company A to mix so that:

- 75*q - 50*(1-q) = 50*q + 25*(1-q) ➔ p = ¾
- The expected payoff for Company B is 43.75 if it mixed 75% in High and 25% in Low.

Repeated games is a good way of learning about competitions, influencing their learning and expectations and achieving a co-operative solution.

2.9. Other games

There is many other examples to fully explain Game Theory, such as games with imperfect information, other situations with uncertainty, multiple equilibriums, etc. Also, there is a lot to talk about signaling, commitment, credibility, uses in actions, among others. However, the examples presented are enough for the reader to have a general understanding about this theory.

In summary, to solve games you can use the following rules (Prasnikar, 2002):

- Rule 1: Look forward, solve the shortest sub-games, and reason back
- Rule 2: Play a dominant strategy if it exists
- Rule 3: Iteratively eliminate dominated strategies
- Rule 4: If there is a unique Nash equilibrium profile, play your Nash equilibrium strategy
- Rule 5: Disregard any Nash equilibrium in which one or players chooses a weakly dominated strategy

3. Game Theory Benefits

Since the reader understood the basic models, examples and concepts, this section presents the benefits of this theory. The benefits section is after the examples on purpose, because it would be more helpful to present some advantages since you understood some concepts, and then you can recall some examples to best abstract some benefits.

As McMillan (1992) stated: “Science is organized knowledge, designed to be efficiently communicated:

- The science of strategic decisions can be learned from a book. By contract, the art of strategic decisions, like any other art, is best learned through experience. By helping us to think systematically through the issues – to understand the logic of the situation – game theory can give us a short cut to what skilled players have learned intuitively from long and costly experience (…)
- Skilled and experienced managers understand principles of strategic decision-making intuitively, but not necessarily in such a way that they can communicate their understanding to other. Game Theory provides a language for expressing these principles (…)
- Real-life strategic situations are often extremely complicated. Game theory provides a model of this complexity. A model is to the real situation what a road map is to the region it represents. The map is a simplification, a deliberately stylized representation that omits some features of the world and highlight others”

Even when all data is available, simplifications tend to be necessary – as in any model-building exercise – before we can apply game theory to a strategic issue. Common simplifications include reducing the number of players in consideration, fixing the values of particular parameter so as to simplify their effects, suppressing uncertainties, and collapsing the timing structure of the situation, often into a one- or two-stage game representation. The tactics place a premium on looking for robust rather than exact solutions and on conducting sensitivity analyses. (Ghemawat, 1999)

As you see, Game Theory also can be used as a model, and there are many advantages of modeling. According to Saloner (1994), “the virtues of formal modeling are, inter alia, that

- It provides an “audit trail” documenting that a coherent explanation for the phenomenon under study can be given;
- It provides a system of logic to root out the flaw in the reasoning in incorrect analyses (as in the case of the maintained output hypothesis);

- It provides a common language which allows related results to be compared, and new results to be build on the foundations laid by earlier models"

Moreover, in its nature, Game Theory has a lot of math. Many researchers focused on finding equilibrium using the logical/math reasoning. Although most of managers would not deep in such details, there are some advantages of finding numbers. As Shubik (1954) mentioned : "The role of mathematics in any study is to provide a language by means of which investigation of phenomena can be carried further than would be possible without the introduction of symbols (…)". The extensive and strategic forms, with a clear relationship between payoffs and strategies, are very useful to make decisions.

In summary, with Game Theory managers can formulate effective strategies, predict the outcome of strategic situations and select or design the best game for them to play.

Even when all moves and payoffs are difficult to be identified, the exercise of putting all information together to simulate best responses generates valuable insights for the decision-making process. According to Ghemawat (1999), "The major contribution that the theory makes it that if forces managers to put themselves into the shoes of other players rather then viewing games solely from the perspective of their own businesses (…) It forces managers to think explicitly about the incentives and likely moves of competitors."

4. Discussion and Limitations

Even presenting all examples and benefits, it is not an easy task to use Game Theory, creating decision trees and making decisions. This section discusses some cautions and difficulty of managers.

4.1. Game Theory as Competitive Advantage

In the previous sections, it was mentioned one of the three uses of Game Theory: explanation. This theory analyzes and explains companies' behavior in competitive interactions.

However, it is necessary a clarification: Game Theory helps manager understand the interactions when players follow game-theoretic principles and assumptions (utility maximization, rationality). So, Game Theory does not show necessarily how companies behave, but how companies should behave using those assumptions. This theory is concerned with how players make decisions when they are aware that their actions affect each other and when each individual takes this into account to maximize payoffs. In fact, Game Theory is helpful to demonstrate the interactions among players who follow ruled-based games.

Is real life like this? Not necessary. In this case, Game Theory acts as a tool for companies to behave differently and to better compete, with good insights to predict strategies. It means to focus on the second and third use of Game Theory: prediction and prescription.

Companies who know Game Theory have advantages over those who do not know, for several reasons. First, it helps think ahead about competitors' reactions. Even being an intuitive task, Game Theory provides better tools than intuition since it structures the reasoning. Second and also important, it prevents choosing bad choices. Modeling all strategies gives a better understanding of all scenarios, and you can clearly see what not to do.

Third, with Game Theory you can understand which situations players don't need to think in competitors' strategies, like in dominant strategies. Also, if competitor does not act as predicted, even though it is possible to take advantage and change strategies dynamically. Moreover, there are all advantages of modeling mentioned in the previous section, such as accelerating experience, creating a common language in your company and providing a audit trail of your scenarios.

In fact, people who don't think in this way have disadvantages since they cannot predict all strategies and they don't fully think strategically, unless they have other tools to structure the same reasoning and scenarios.

4.2 The difficult part for managers

Managers might think that, when all possible moves and payoffs are given in a matrix or game tree, it is very easy to play, since Game Theory teaches how to solve using backward induction, Nash equilibrium, mixed strategy, etc. Taking decision in a structured situation – game tree with defined players, strategies and payoffs – is very straight forward.

Until now, examples were very simple and they focused on how to solve a problem given that you and all players had the same game tree and payoffs. We focused on how Game Theory helps people solve a problem, not to elaborate the question (although it also helps elaborate the problem – we will cover this later). In fact, the most difficult part for managers is to design the game – finding payoffs and all strategies.

To help in this issue, it is important to understand that the main limitations of game theory lie exactly in its own assumptions[13]. Remember that Game Theory assumes that:

- all players want to maximize their own utilities and reason as rational players to achieve their goals (utility maximization and rationality)
- all players know all possible moves and payoffs of each other (common knowledge)

Real life is more complex and fully implementation requires (1) enough amounts of data and (2) knowledge of others. In fact, it is assumed that you know your market and competitions. Although Game Theory can offer some insights in those requirements, it does not substitute business experience, other approaches and business tools. Knowledge of competition is a fair assumption because, regardless Game Theory, managers should know it to compete and survive. If managers use other theories, they still need to have this knowledge.

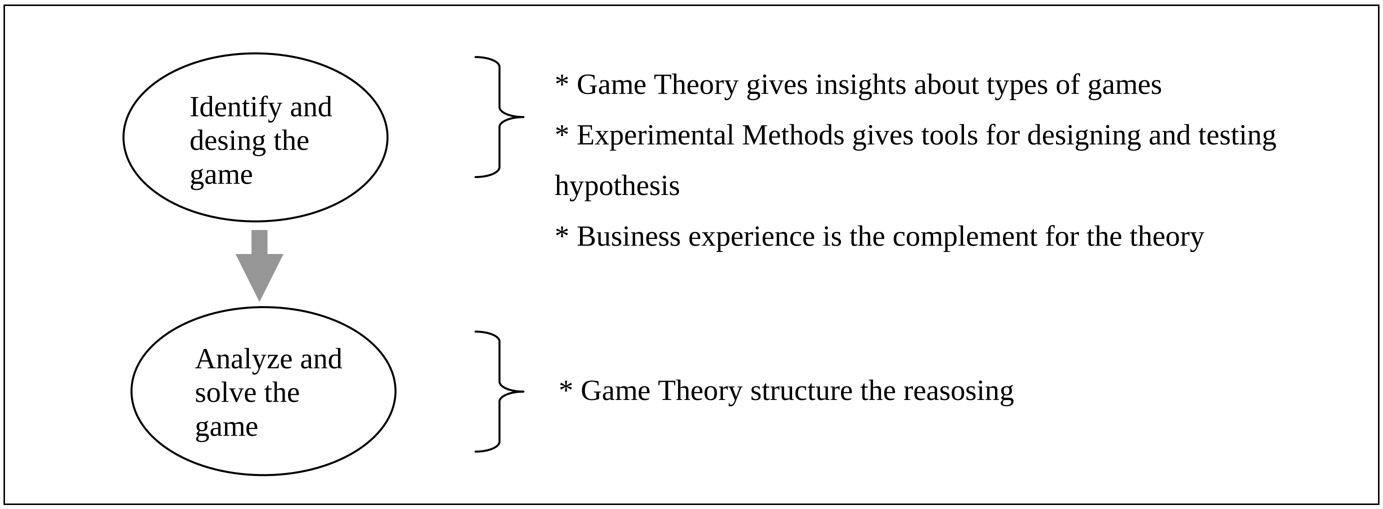
One example is the NPV (net present value) calculation. Finance courses teach the concept of using NPV in the decision making process about investment, etc. It teaches the formulas to calculate the NPV. However, in Finance courses, future cash flows (the input) are given, and, if they are not, it is given numbers of revenues and costs to calculate them. However, the most difficult part of managers in real life is to find the future cash flows. In this case, all company is involved – it depend on the marketing strategy, operations cost, and many other factors. Without those inputs, there are no cash flows, and without cash flows, NPV formula is useless.

Similarly to NPV issue, where it is assumed that you know the cash flows given other inputs, Game Theory also needs extra inputs from managers, such as correct payoffs and understanding of competitors' behaviors and

[13] It is my idea and also is the table – I haven't seen in any literature

alternatives. In NPV calculation, sometimes approximate numbers are enough to identify is the NPV is positive or negative. In Game Theory, approximate numbers are enough to identify which payoff is bigger and smaller, better and worse.

Picture 17 shows the two main elements to fully compete and have competitive advantage. You need to know how to identify and design the game correctly, and then analyze and solve with your best strategy.



Picture 17

It is important to understand where Game Theory offer more help and where managers need to do their part in the reasoning. Game Theory has a great influence in the reasoning of finding best strategies. However, it has some but limited influence in helping people find payoffs and all possible strategies.

Suppose you have all information you need and knowledge is not a constraint – you have a perfect game tree, all strategies and accurate payoffs. Then there are other questions. Do your competitor have the same schematic representation, alternatives and payoffs? How do you know if your rival is playing the same game? So, you need also know his capabilities in facing the same problem.

It is true models are very specific and the predicted behavior depends on if your competitor follows your model, and in a rational way. Saloner (1994) mentioned: “The degree of rationality assumed in game-theoretic models is often much greater than in other economic models. In game-theoretical models each firm’s optimal action depends on what it believes its rivals will do. In order to decide what to do itself, the firm must put in its rival’s shoes and analyze the situation from its rival perspective. Then, the analysis therefore requires assumptions about the rival’s rationality, as well as the assessment of the rival’s belief about one’s own rationality, and so on. These assumptions are particularly striking in a field like strategic management, which tolerates a wide variety of behavioral assumptions”

I believe that those varieties of behaviors are connected to the payoff structure of the game and preferences of each player, and that players are still rational if we defined rationality as pursuing their utility maximization. Next section shows the importance of payoffs in designing a game, and some insights for managers don’t make some mistakes.

4.3. Influence of payoffs in the decision-making process

"An agent is, by definition, an entity with *goals and preferences*. Game theorists, like economists and philosophers studying rational decision-making, describe these by means of an abstract concept called *utility*. That is, when we say that an agent acts so as to maximize her utility, we mean by 'utility' simply whatever it is that the agent's behavior suggests her to consistently desire" (Ross, 1997).

This is the most critical issue to understand and use Game Theory. In general, a game is partly *defined* by the payoffs assigned to the players. A utility function for a player is supposed to represent *everything that player cares about*, which may be anything at all. As we have described the situation of our prisoners they do indeed care only about their own relative prison sentences, but there is nothing essential in this. What makes a game an instance of the Prisoner's Dilemma is strictly and only its payoff structure. To return to our prisoners, suppose that, contrary to our assumptions, they *do* value each other's well-being as well as their own. In that case, this must be reflected in their utility functions, and hence in their payoffs. If their payoff structures are changed, they will no longer be in a Prisoner's Dilemma (Ross, 1997).

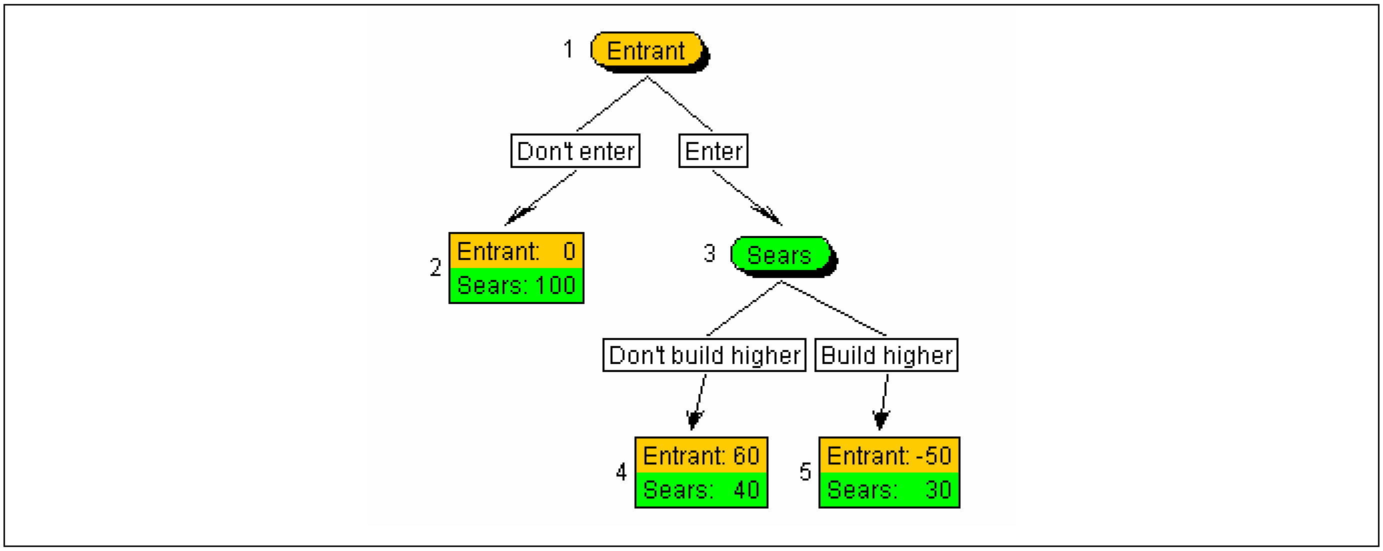
So, it is important to capture:

- the right value of the payoffs (the number itself)
- the nature of the payoff (profit, market share, long term, short term, any sort of ranking, etc)

As a result, it is important to know the competitors' motivations and characteristics. Game Theory assumes the common knowledge and utility maximization, but indeed it is not an easy task. The following examples demonstrate how different payoffs change completely the nature and results of a game.

4.3.1. Understanding the real utilities and competitors' behavior

Let's return to our first example, the Sears towers. The Entrant will choose Enter because it knows that Sears will choose Don't Build Higher since it provides a better payoff for sears (40 in node 4 against 30 in node 5). Picture 18 shows the extensive form again.



Picture 18

However, a question that a manager, in the role of Entrant, could have is: "I calculated the payoffs based on my information about costs and revenues. But did Sears do the exact same calculation? How should I know if my 40 is its 40?" In fact, it is a pertinent question. In our Game Theory concepts, we assume that companies have common knowledge to get the same payoffs. In real life, companies don't exchange game trees to check if they have the same game before taking decisions.

If Sears thinks that its payoff of building a higher build is not 30, but more then 40, it changes completely the game and your decision, since you will get -50 and it is better not to enter. There are some reasons why Sears would choose Build Higher.

First, by mistake, i.e., the payoff is indeed 30 (you calculated correctly) and Sears calculated wrongly. In this case, Sears do not maximize its return, and you lose because of Sears' error.

Second, because the payoff is indeed more then 40 and Sears calculated it correctly. So, Sears maximize its payoffs and you, who calculated wrongly, lose.

Third, because Sears see extra benefits, and this changes the utilities of the game. For any reason, even though the return is 40 (for example, money), Sears see more advantages in building a higher tower. For example, Sears can look some benefits in long run, or it has some information that you don't have. Also, because Sears wants to harm you since your payoff is negative and it can have lower payoff just to keep you out of business and signal the markets to prevent against other competitors. Again, it means that the utilities of building a higher tower is greater.

There are some advices for the manager to not make those mistakes.

- In the first situation, when Sears act by mistake, you need to pay attention to your competitors' ability of make decisions. Some argue that it is difficult to play against an irrational player. If you indeed know your competitors, you should know their behaviors. Since you know some possibilities of a non utility maximization action, you can use this knowledge in your advantage and change your decision. Therefore, it is not an irrationality issue, but it is a matter of how predictable is your rival.
- In the second situation, when you calculated wrongly the payoffs, it is not a Game Theory's responsibility. Moreover, if you don't have data or don't know how to use it to forecast consequences, it is indeed very hard to compete, regardless game-theoretic approaches.
- In the third situation, you should capture the real utilities and preferences that Sears care about and incorporate in the payoffs. Again, it assumes that you know your competitors. To correctly play the game and make decisions, managers must not only calculate obvious market numbers, but understand all motivations of their rivals.

Remember that Game Theory requires that players make rational choices based on the payoff structure. If the payoff is different or incorrect, it is not rationality issue. "Because of the interdependence of players, a rational decision in a game is that one based on a prediction of other's responses. By putting yourself in the other's shoes and predicting what action the other person will choose, you can decide your own best action" (McMillan, 1992).

Some people could argue that individual are not rational and then Game Theory is useless. There are some comments about this. Apparent irrational people might not have clear preferences in their payoff structure. It is not the case of companies that care about their profits. Companies are more rational then individuals and they don't want to lose money or market. If you really know your competitors (after all, you know their CEO and all staff) you have information if they will do the best guess to play the right game.

Some also could argue that companies can act irrationally as part of the game just to make you confused. Now, as mentioned, it is an issue of predictability. In fact, your competitor wants to be unpredictable (as you would like to do sometimes). Your objective then is to measure how unpredictable he is, and the consequences of each behavior. You can give weight and percentages in each likely action and find some expected payoffs, measuring the impacts to make a decision.

It is more common in repeated games, so you can learn, during the game, your competitors' behavior. You can find the right mix to also change strategies. Moreover, if he does not play as predicted, take advantage – you don't need to follow the predicted equilibrium if you can earn more with your competitor's mistake.

4.3.2. Understanding the composition of payoffs – second example

Even if you capture the correct economic measure of your payoff (profit or revenue, for example), you also need to understand what metrics your competitor values to make a decision. For example, is it in absolute or relative number?

Consider a country where retail banking is a very competitive industry and has been dominated by two major players, namely ABC and JKL. To stay ahead of the competition, it is necessary to be big by expanding operating in the whole country. Then, the best strategy to do so is to increase the numbers of clients and asset by absorbing other banks.

Three small banks announced that they are going be sold, one in each month. The banks X, Y and Z will be already to be sold in month 1, 2 and 3, respectively. ABC and JKL has cash to buy all three banks, but there are some characteristics and constrains in this competition.

a) Rule 1. The negotiations had already begun. ABC has an advanced negotiation with X, while JKL has an advanced negotiation with Y and Z. So, it is natural that the sellers have preferences to whom to sell, and the players know that. If the preferential buyer refuses, other will have the opportunity to buy. If the second refuses, both cannot try to buy later because there are more potential buyers. The sequence and preferences are:

Month	Seller	Preferential buyer – can buy first	Secondary – Can buy if the first refuses
1	X	ABC	JKL
2	Y	JKL	ABC
3	Z	JKL	ABC

b) Rule 2. Because of the amount of employees involved in the process of acquisition, it is not viable for any of them to buy 2 banks in consecutive months. For example, JKL cannot buy banks Y and Z. With the same reasoning, ABC cannot buy X and Y, even if JKL refuse to buy Y.

c) Rule 3. The payoff is based in the assets of the new institution. However, it is not calculated with the sum of the assets of acquisition, but with the difference between them. The payoff of ABC is (new assets of ABC) – (new assets of JKL) and payoff of JKL is (new assets of JKL) – (new assets of ABC).

For example, if ABC bought X and Z (assets=8+9) and JKL acquired Y (asset=10), the payoff of ABC is 7 (=17-10) and JKL is –7 (=10-17). It means that ABC is +7 units of asset in front of JKL, and the JKL is –7 units behind ABC.

Looking at the payoff from the mathematical perspective, the objective of each player is to find the best strategy to maximize its own payoff. The game consists of 2 players (ABC and JKL) who make their decisions independently about whether or not they will buy the bank of the month. If the preferential quit, the secondary can buy the bank in the respective month.

Given perfect information, they know the decision of each other and the rational outcome is predictable. They must follow the rules 1 and 2, namely, (1) the preference of the month and (2) if they had already bought a bank in the previous month.

The players, the options and the payoffs are in the picture 19. Due to constraints of the scenario, those options contradicting established rules would not appear in the tree. For example: after ABC has bought X, if JKL chooses "not buy Y", ABC does not have the option to buy Y in that month.

The assets that they gain by buying the banks are as follows:

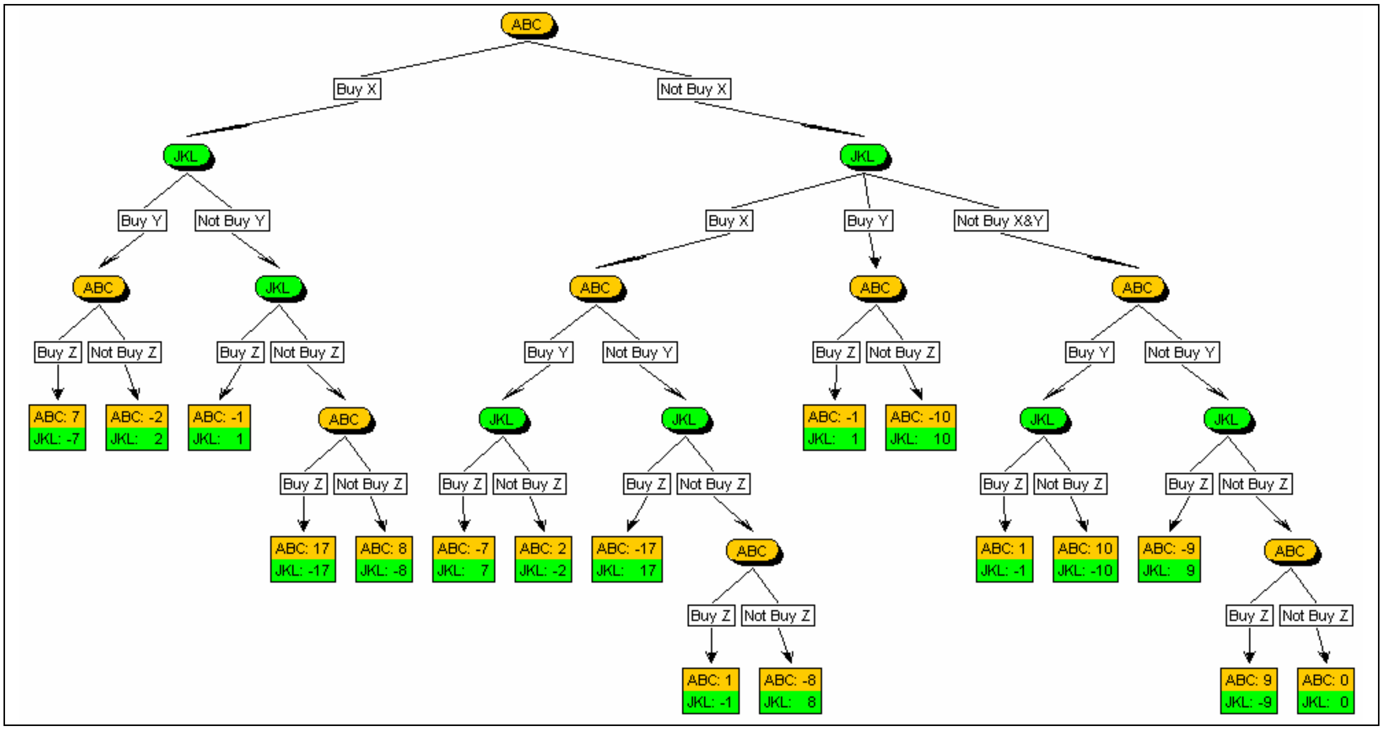
Bank	Assets
X	8
Y	10
Z	9

The rational to find payoff - due to these constrains, there is 17 possible outcomes:

#	Nodes		Payoff	Bank X	Bank Y	Bank Z	New Assets ABC	New Assets JKL	**Payoff ABC**	**Payoff Brad**
1	4		9	I	B	I	17	10	**7**	**-7**
2			10	I	B		8	10	**-2**	**2**
3	5		11	I		B	8	9	**-1**	**1**
4		12	19	I		I	17	0	**17**	**-17**
5			20	I			8	0	**8**	**-8**
6	6	13	21	B	I	B	10	17	**-7**	**7**
7			22	B	I		10	8	**2**	**-2**
8		14	23	B		B	0	17	**-17**	**17**
9			29	B		I	9	8	**1**	**-1**
10			30	B			0	8	**-8**	**8**
11	7		15		B	I	9	10	**-1**	**1**
12			16		B		0	10	**-10**	**10**
13	8	17	25		I	B	10	9	**1**	**-1**
14			26		I		10	0	**10**	**-10**
15		18	27			B	0	9	**-9**	**9**

16			31			I	9	0	**9**	**-9**
17			32				0	0	**0**	**0**

I = ABC buy; B = JKL buy.
Payoff ABC = (new assets ABC) – (new assets JKL)
Payoff JKL = (new assets JKL) – (new assets ABC)



Picture 19

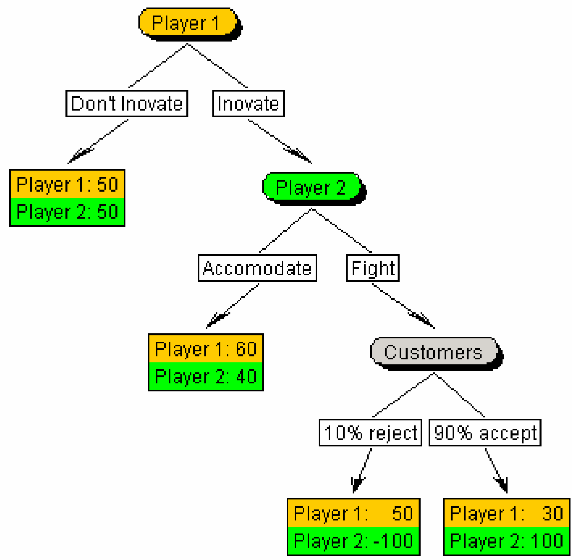
To find the equilibrium, it is just to use backward induction in the whole tree and it is very easy. However, the solution here is not important.

The goal with this example is to show that:

- Players' payoffs are based on the difference of assets, not the absolute value of asset that they purchase. It is important to be ahead from competition. Therefore, this preference was capture in the payoff structure. If you were to solve this game just looking the assets you acquire, the solution would be different.
- You need to understand the constraints and capabilities of the market and competitors. For example, those companies have money to buy all banks, but they don't have enough staff to operate the acquisition, otherwise they could lose money in a poor administration. So, the design of strategies was modeled considering this situation. However, if your competitor has some capabilities to purchase all banks in consecutive months and you don't (or vice-versa), the design of the game would be totally different, as well the solution.

4.3.3. Considering managerial styles

You need also to consider the managerial styles of your competitors and design your payoffs accordingly. Let's take the example below. You are Player 1 who is considering include an innovation in one product that has a strong competitor with a similar product. If you don't innovate, the current payoffs are 50 for each one. If you innovate, Player 2 has the option to accommodate and do nothing, what increase your payoff to 60, and decrease its for 40. If it chooses to fight and launch its own innovation, there is a risk of 10% that customers reject Player 2's product (it loses -100), and 90% of them to accept (it gets 100). The payoffs in both situations are described in the picture 20.



Picture 20

Using game-theoretic principles, we calculate the expect payoff in the customer branch. Then, the expected payoff of Player 2 is 80 (=10% x -100 + 90% x 100), and yours is 32 (=10% x 50 + 90% x 30). In this case, Player 2 prefers to fight since 80 is greater then 40, in the accommodate option. As you will get 32, you prefer not to innovate and keep the payoff of 50.

We see that 90% is a high probability for Player 2 to get a payoff of 100 and you 30. Now, what if Player 2, being risk averse, does not want to take the risk of losing 100? Maybe it chooses to accommodate because it prefers to get 40 for sure. In this case, it would be better you innovate.

Should you rely on the payoff structure or follow intuition? In fact, it is not the correct question. If you know that your competitor is extremely risk averse, the Player 2's payoff is not like the one described. If it prefers to accommodate than to take a risk of fighting, it means that the utilities for accommodate is higher regardless the economic payoff of the expected value. Knowing that, you need to adapt your payoffs to represent the real behavior.

Player 2's behavior is not necessary irrational and it is not against game-theoretic concepts since it has other preferences that should be captured in the game tree. The art of Game Theory is in finding those preferences to better compete. Notice that Game Theory continue being valid. It helps to map situations in with intuition is harder to use. The correct should be to incorporate the risk averse behavior in the modeling.

4.4. Designing the correct game

In a price war, for example, some companies have more capabilities to fight and reduce price. Others cannot do it because of their cost structure, risk averse behavior of managers or other motives. Computational tools do not substitute the intuition and knowledge of managers to design the correct game, preferences and payoffs. In fact, managers should help Game Theory to predict the correct answers. Game Theory shows that, since you predict all possible moves, you can act accordingly.

Game Theory helps organize the manager's thoughts to create and analyze an efficient game. Two examples below show how managers designed correctly and wrongly some games. You should be aware of such examples of success and failures in designing the correct game.

4.4.1. Example 1 – Kiwi Airlines[14]

When a new player enters with a price lower then the incumbent's, the incumbent has only two effective responses: match the newcomer's price across the board or stand pat and give up share. Kiwi International Air Lines is a 1992 start-up founded by former Eastern Air Lines pilots who were grounded after Eastern went bankrupt. Kiwi engineered a cost advantage from its employee ownership and its use of leased planes. But it had lower name recognition and a more limited flight schedule than the major carriers. What did it do?

It went for low prices and limited capacity. Why? When an entrant adopts this strategy, where it makes any money depends on how the incumbent responds. The incumbent can recapture its lost market by coming down to match the entrant's price, or it can give up, say 10% of market. For the incumbent, giving up 10% of share is usually better then sacrificing its profit margin. But the entrant cannot be too greedy; if it tries to take away too much of the market, the incumbent will fight to regain share, even giving up some margin. So, only when the entrant limits its capacity does the incumbent accommodate and the entrant can make money.

That is what happened and Kiwi made money by staying away of large carriers and making sure they understood that Kiwi pose no threat. Kiwi intended to capture, at most, only 10% share, or no more than four flights per day. To craft the right choice of capacity and price, Kiwi had to put itself in the shoes of the major airlines to ensure that they would have a greater incentive to accommodate rather then fight. It shows how Kiwi's managers understood their competition and designed the correct game.

[14] Example from Brandernburger, 1995 with some modifications to fit in this paper

4.4.2. Example 2 – Holland Sweetener Company[15]

NutraSweet, a low-calorie sweetener used in soft drinks as Diet Coke and Diet Pepsi, generated 70% gross margin for Monsanto. Such profits usually attract others to enter the market, but NutraSweet was protected by patents in Europe until 1987 and in the United States until 1992.

With Coke's blessing, an entrant, the Holland Sweetener Company, build an aspartame plant in Europe in 1985 in anticipation of the patent expiration. As HSC attacked the European market, Monsanto fought back aggressively. It used deep price cuts and contractual relationships with customers to deny HSC enter the market. Then, HSC was looking forward to moving the war into the United States.

However, the war ended before it began. Just prior to the US patent expiration, both Coke and Pepsi signed new long-term contracts with Monsanto. It seems that Coke and Pepsi didn't seize the opportunity of competition between suppliers. In fact, neither Coke nor Pepsi even had any real desire to switch over to generic aspartame. Neither company wanted to be the first to take the NutraSweet logo off the can and create a perception that is was changing the flavor of its drinks, since NutraSweet had already built a reputation for safety and good taste.

In the end, what Coke and Pepsi really wanted was to get the same old NutraSweet at a much better price. HSC should have recognized that Coke and Pepsi would have paid a high price up front to make the aspartame market competitive. HSC didn't design the correct game. Coke and Pepsi played correctly. And Monsanto did well to create a brand identify and a cost advantage, minimizing the negative effects of entry by a generic brand.

[15] Example from Brandernburger, 1995 with some modifications to fit in this paper

4.5. What the literature offers to managers

The reader needs to understand that this paper does not cover all possible literature available in the market. However, I researched the most current popular books, including those used in MBA schools, and also articles in business magazines. I believe that those are the main sources of a normal manager who wants to keep in touch with Game Theory.

The main problem for managers is that few books explain a complete use of Game Theory, with real examples. As you noticed, the examples of section 3 and 4 (although they represent just 10% of all game-theoretic theory) are very simple and just explain the reasoning of Game Theory. Nevertheless, they represent about what 90% of Game Theory books cover. Many other books explain with more details some business situation, but they don't show a decision tree or payoffs. When some mention the payoffs, they don't explain from where they come from. Other articles just mention the importance of playing the right game and the strategic thinking.

For sure all books and examples are important to create the big picture to managers and most of the benefits covered in the section 2 can be achieved with such literature. In summary, those are valid and help managers.

Looking in many literatures, there are five types of examples that I found in books and articles:

- Example Type 1 = examples without the numbers, decision trees or payoff matrix. They focus more in the strategic thinking using Game Theory concepts and business environment
- Example Type 2 = examples with a business meaning, complete mathematical model description and solution, but the payoff are given
- Example Type 3 = variation of type 2, but with much more simplifications in the explanations
- Example Type 4 = examples with only mathematical hypothesis (ex: players A, B; actions C, D)
- Example Type 5 = examples of very sophisticated payoff calculations with no simplifications of model to be understood by managers

The examples given in this paper are classified as type 2, 3 and 4. As mentioned, managers who read all these 5 types might feel that it is missing something to go to their office and applied Game Theory. If fact, there a few examples in the literature that combines: (a) business set and explanations, (b) payoff calculations and (c) model's design and solutions.

While many authors advocate that Game Theory is an important tool, there are few complete real examples. Why? As Ghemawat (1997) stated "Practical applications of game theory to business remain so novel as to be the subject of considerable excitement". He also mentioned that there is a "missing empirical link: game-theoretic models have not lent themselves to conventional large-samples tests to an extent that would, if the results were positive, persuade researchers in business".

4.5.1. A good example – Ware Medical Corporation[16]

Below is a good example of a case study with a scenario description and numbers to find a payoff.

Louis Richardson, a polymer chemist at Ware Medical Corporation routinely scanned the contents of the new Federal *Bulletin of Materials every* month, but the June issue this year brought him quite a shock when he saw the announcement of FR 8420. FR 8420 was a new acrylic polymer with silicate side chains that had just been developed by NASA scientists, as a byproduct of their research on materials for use in the space shuttle. Richardson immediately recognized that this new discovery could threaten Ware's position in the market for dental materials.

Ten years ago, after six years of research, Ware Medical Corporation had received a patent for a new method of making translucent composite materials that combined glass particles in a matrix of plastic. Before that time, prosthetic dental restoratives had been predominantly made of plastic and porcelain. Porcelain had the advantage of being more resistant to abrasion, but it was also more brittle than plastic, so that neither material was clearly superior for all purposes. Ware's composite material, which was marketed under the name of Dentosite, combined the best properties of both materials. Dental laboratories and manufacturers of artificial teeth were quick to recognize the advantages of Dentosite. According to a study that was published four years ago by Data Research Corporation, Dentosite had captured a 60 percent share in the market for materials used in dental prosthetics.

National Dental Corporation was the largest supplier of materials for dental prosthetics before Dentosite. Five years ago, National had entered into lengthy negotiations for the right to manufacture and sell composite materials using Ware's patent process. At times, it had seemed that an agreement was imminent; but negotiations broke down two years ago and National initiated a lawsuit to contest Ware's patent. Although the suit was still pending, Ware's lawyers were confident of winning.

[16] From Roger B. Myerson, Kellogg School of Management, Management and Decision Science Department, Northwestern University

Charles Piper was the vice president of Ware responsible for the dental products group. On June 21, three days after Richardson read the announcement of FR 8420, Piper held a meeting with Louis Richardson and Benjamin Gretter, who had general responsibility for the Dentosite product.

Richardson began the discussion. "Our Dentosite material has held a unique position in the market essentially because of our patented process for preparing the glass particles to bond to plastic. However, such preparations could be entirely omitted if the usual plastic materials were replaced by this new material FR 8420, because it bonds directly to glass.

Of course, it would take some time to develop a new composite with FR 8420 that could serve as a dental material. The main problem is that the glassplastic bond that one could get with FR 8420 would not be as strong as what we get with our process. The only way to overcome that problem would be to try to use a fibrous glass component. I figure that there is a 50% chance that an acceptable translucent composite is feasible using fibers with FR 8420. So if we are lucky, it might not be feasible, but we cannot count on such luck. It seems to me that our best bet is to work on developing a translucent fibrous composite ourselves. If the technique is feasible, then we would have just as good a chance as National of being the first to prove it. Then, if we developed it first, we could extend our patent protection to this technique and prevent any competitor from making fibrous composites with FR 8420."

"Lou and I have gone over the numbers to justify this plan," Gretter said. "If the technique is feasible at all, it should take two years of work, for us or for National, to develop an acceptable fibrous composite using FR 8420. We would have to budget about $500,000 per year to the project. National would probably need to spend more, about $1 million per year for two years, because their goal would be to develop a product ready for mass production, whereas we are just trying to prove feasibility to get the patent."

"Our current patent expires seven years from now, after which anybody can make composites like Dentosite using our current techniques," Gretter continued. "So this alternative fibrous technology would be only valuable to National during the next seven years. That means that they really would need to get this new composite developed and into production within two years or it is just not worthwhile for them. On the other hand, if they do develop an acceptable fibrous composite in two years (and if we do not stop them by getting a patent on the process first), then during the last five years of our patent we will probably lose about onehalf our market for Dentosite to them. From their point of view, it must look pretty risky, and I cannot imagine that anybody else besides National Dental Corporation would be willing to even consider trying to develop this technology."

According to the projections of the Data Research study, demand for our composite should be between $15 million and $20 million per year over the next seven years. Our profit margin has been 20% of sales of Dentosite, and I expect that National would follow a similar pricing policy. They would not need to start a price war to take market share from us in this field, once they had a product to sell. So if we project sales at $17.5 million per year and use a 10% discount rate, the present discounted value of profits from onehalf the market for Dentosite during the period between two and seven years from now is $6.0 million."

After Gretter finished, Piper made a few notes and tried to summarize the situation. "Our whole problem seems to depend on what National does," he said. "It is foolish to spend money to develop a technology that we do not want to use if National is not trying to develop it. On the other hand, if National is trying to develop this technology, then we cannot afford to drop out of the race. So it all depends on how the people at National see this situation. Do you think that they see it as you have just described? Is there anything that we know that they do not know?"

"Everything that we have discussed so far is commonly known in the industry," Gretter replied. "Certainly National has people who check the *Federal Bulletin* every month, just as we do. If they have not noticed FR 8420 yet, they certainly will soon. Except for a few minor details, we probably have the same information about the technology and economics of the situation. I used Data Research's expected projections precisely because they are what National would be considering. Actually, our annual sales have been around $16.0 million per year, and that is probably a better estimate of future annual sales than $17.5 million. But that does not look like a significant distinction, in view of all the other uncertainties."

To solve this game, we could say summarize (Prasnikar, 2002)

- A new material FR 8420 that was developed by NASA.
- 10 years ago Ware received a patent for Dentosite that has captured 60 percent share in the market
- National was the largest supplier of material for dental prosthetics before Dentosite.
- If Ware develops a new composite with FT8420 it will be a substitute for Dentosite
- If the technique is feasible then Ware would have just as good chance as National of being the first to prove it.
- If Ware develops it first they could extend the patent protection to these techniques and prevent any competitors.
- National entry cost: $ 1,000 million; Ware entry cost: $ 500 million
- Range of possible future annual sales: $15 - $20 million; using the average :$ 17.5 million
- Ware profit margin: 20%

- Probability of process is feasible: 50%
- Ware change of winning the rage: 50%
- Ware market share if National enters: 50%
- Discount rate: 10%
- Value of $1 in years 1 and 2: $1.9; value of $1 in years 3 to 7: $3.4

We will skip the solution here because the math is a little complex for the purpose of this paper. Basically, you need to find a payoff matrix of NPV of future profits if Ware and National enter or not. However, this example shows a good description for managers to understand the scenario and design the game using the information provided.

4.5.2. Some suggestions to improve the managers' ability of using Game Theory

As mentioned, many books provide good insights for managers to use Game Theory. Probably, new publications with more practical (fictitious or real) complete examples will be published, such as Ware Medical Corporation or the book "Games Businesses Play – Cases and Models" (Ghemawat, 1997).

Theories tend shift from academia to consulting companies. Those companies are a good way to transmit practical knowledge to businesspeople[17].

Game Theory is a specific course in many business schools of MBA. In the future, more and more managers from those schools will have a better understanding of Game Theory, and then they can put their knowledge in practice. Also, business students can use experimental methods to test and design games. As mentioned, software from ComLabGames[18] are excellent learning tools. Moreover, students can test hypothesis about people behaviors by simulating games among several subjects. .

Training program in corporations is also a good opportunity to propagate Game Theory in practice. For the company, it is essential to create a common language in strategy and develop the managers' skills in competitive

[17] The site http://www.gametheory.net presents a list of "Game Theory Consulting" companies. With a quick look, I could notice that the listed companies use Game Theory as part of their business consulting in strategy. Some times, even the name Game Theory is mentioned. Buy looking the websites of big consulting companies, Game Theory rarely appears in the headlines or when it is searched as a keyword.

[18] See http://www.cmu.edu/comlabgames and Appendix B for more details

situations. All benefits from section 3 can easy be achieved using Game Theory as discipline. As an open software, ComLabGames can be used in companies to train managers, simulate environments and test behaviors.

In fact, Game Theory must be tested and published more and more. As Saloner (1994) mentioned, “only by applying game-theoretical modeling to issues in strategic management will be able to assess its usefulness to the field”.

4.6. When and how to apply Game Theory

As the given examples, Game Theory can be used by companies to make decisions, such as:

- capacity expansion and reduction
- entry and entry-deterrence
- marketing and advertising expenses
- price settings
- bidding
- negotiations

"Game-theoretic thinking is most helpful when there are only few players whose actions or reactions really matter for a particular issue. The number of players under consideration can also sometimes be reduced by aggregating players with similar economics and objectives" (Ghemawat, 1997).

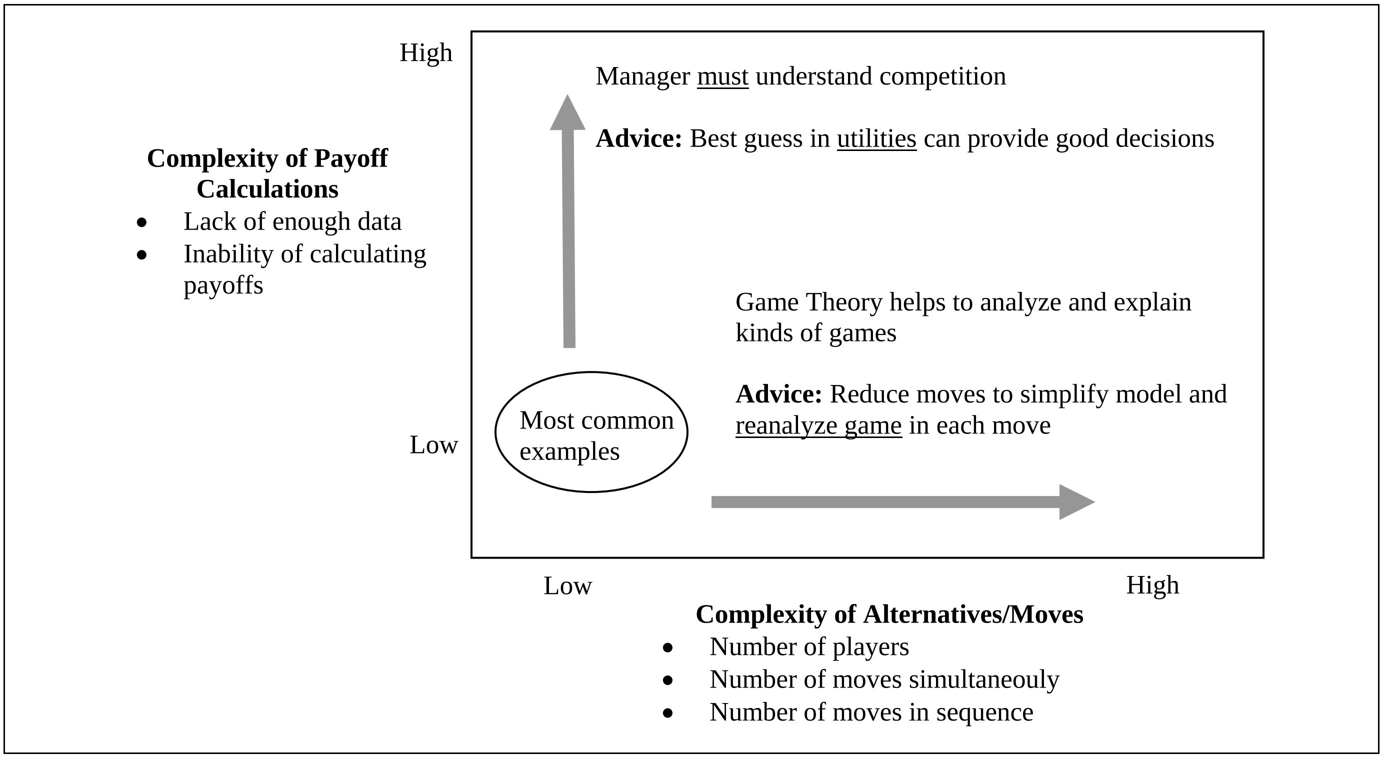
According to Ghemawat (1997), "Several additional factors also influence the benefits for game-theoretic analysis:

- Identifiable players, relatively clear-cut options for them, and good data sources all facilitate the task of mapping actions into payoffs.
- Players' familiarity with one another and their repeated interactions among them increase the likehood that they will actually reason or grope their way to game theoretic equilibrium, enhancing its usefulness as a reference point.
- Attractive structural features – in addition to the presence of a small number of players – expand the scope of game-theoretic analysis, enabling it to generate counterintuitive "cooperative" insights.
- Finally, an organization's embrace of an analytical culture can ease its assimilation of game theoretic techniques and analysis. A sophisticated financial analysis is usually a complement to, rather than a substitute for, game theory in improving economics outcomes"

So, Game Theory is best applied when there are (1) few players, (2) clear and few options to move, (3) enough amount of data and (4) knowledge of others.

We have mentioned that the difficult part for managers is to calculate payoffs and design the game. The picture below presents a schematic representation about when Game Theory is more common and what managers should do to take advantage in complex situations.

Complexity of payoffs can be summarized by the lack of enough data or the inability of calculating the right payoffs or utilities. Complexity of alternatives (or of modeling) is represented by the large number of players, or numbers of simultaneous or sequential moves.



Picture 21

To tackle games with high complexity of payoffs, we have already mentioned that it is an assumption that managers know their industry and are able to identify the correct numbers. Even though it is very hard, simplifications can (and must) be done to make decisions. As any model, managers should have the sensibility to bet in their best guesses in the utilities that are important. Moreover, to make a decision, it is not necessary to find the exact number – it is enough to understand if one option has a bigger or smaller payoff; sometimes, it is enough just to know what is better and worse.

To address games with high complexity of alternative, Game Theory provides a framework to analyze if the game is a one-shot game or repeated game, for example, and how to find a optimum mixed strategy or expected utilities. Even if you recognize that there are several alternatives in sequence and that the game is very long, you can simplify it and redesign it in each move; you must need to change your strategy dynamically.

In defining your game and simplification, “often there will be no unique answer: it will depend on how much detail you want to go into. In analyzing a situation as a game, you face trade-off between realist complexity and comprehensible simplicity” (McMillan, 1992).

While there are not many examples available to teach managers how to address some debatable issues – such as complexity, rationality, common knowledge, payoff calculations and strategies design – some advices and hints provided from Game Theory and modeling are very helpful to increase competitiveness and make good decisions.

As Saloner (1994) mentioned:
“I have argued that in order to evaluate game-theoretic modeling it is important to consider the goals of the analysis. I have argued that one’s appraisal of the rationality assumption and of the usefulness of the models depends on whether one takes a literal or metaphorical view of microeconomic-style modeling. (…) I have argued that literal game-theoretic models are likely to have only a limited role. In particular, game-theoretic models have not proven very useful, nor are they likely to, for providing precise descriptions for managerial behavior. Rather, the role of game-theoretic models is to produce insights from well-reasoned models.”

Conclusion

"Game theory is the study of the ways in which *strategic interactions* among *rational players* produce *outcomes* with respect to the *preferences* (or *utilities*) of those players, none of which might have been intended by any of them. The meaning of this statement will not be clear to the non-expert until each of the italicized words and phrases has been explained and featured in some examples" (Ross, 1997).

With this paper, I hope that you have understood those italicized words and Game Theory concepts. Moreover, you now might have some insights about competitive interactions and pay attention on some crucial details in order to not make mistakes.

There is a lot to improve to make Game Theory more popular for managers. Meanwhile, you should use Game Theory as informed judgment to accelerate experience. It can help you simplify the world and provide advices to be a better decision-maker in your competitive world.

So remember to think about the following:

- Identify the players for your specific decision-making situation (your game). Are all direct competitors or also customers, suppliers, etc? How many are them? Can you simplify and reduce them for your analysis?
- What is your ultimate goal? Do you have intermediate steps to achieve this objective?
- How can you measure your objective? What are your utilities and preferences?
- Do your competitors have the same goal? Do you know their real utilities? Do you and your competitors have the same payoff structure?
- What are your options to maximize your payoff and achieve your goal? What are your competitors' options? Do they have different strategies than you?
- Do you and your competitors have the same information and knowledge to figure out all possible strategies and utilities? Are actions and payoffs consistent among themselves? Are you and your competitors playing the same game?
- Do you know your competitors' capabilities to figure out what strategies are feasible? Do you their managerial styles and consequent changes in preferences and options? Do your competitors know yours?
- What is the nature of your game? One-shot game or repeated games? Do you know your mixed strategy? Is there any uncertainty to calculate? Your game is sequential or simultaneous? Is there any dominant or dominated strategy, or Nash equilibrium? Can you signal some credible commitment or threat?
- If you were you competitor, what you would do?

Of course, those are not easy questions. However, analyzing in a structure approach to make decision is more helpful then acting without strategy. As you could understand, Game Theory provides a new set of tools for understanding strategy. It has much to offer management strategy, both as a guide to analyzing how managers compete and as a method for devising competitive actions (Spulber, 1997)

And as McMillan (1992) mentioned: “Managers are continually making strategic decisions (…) Game Theory provides a tool for thinking about how deals are shaped: a systematic way of thinking about questions of strategy (…) It is a way of economizing on experience: Game Theory makes it possible to grasp the principles of strategic thinking”.

So, with this paper, I hope to have helped you to understand Game Theory and its principles[19].

[19] Obs: all examples and discussions made in this paper were related to companies that wants maximize their objectives (profits, revenues, market share, etc). However, it is not the only application of Game Theory. People who know this theory can use it to analyze and predict actions in several environments, such as: (1) games: cards, tic-tac-toe, (2) sports, (3) relationships - such as wife-husband discussions, job-seeker and employer negotiation, (4) government relationships - cold war is a rich example and (5) contracts with suppliers and auctions.

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Appendix A – Chronology of Game Theory

From http://www.econ.canterbury.ac.nz/hist.htm, by Paul Walker (2001)

0-500AD

The Babylonian Talmud is the compilation of ancient law and tradition set down during the first five centuries A.D. which serves as the basis of Jewish religious, criminal and civil law. One problem discussed in the Talmud is the so called marriage contract problem: a man has three wives whose marriage contracts specify that in the case of this death they receive 100, 200 and 300 respectively. The Talmud gives apparently contradictory recommendations. Where the man dies leaving an estate of only 100, the Talmud recommends equal division. However, if the estate is worth 300 it recommends proportional division (50,100,150), while for an estate of 200, its recommendation of (50,75,75) is a complete mystery. This particular Mishna has baffled Talmudic scholars for two millennia. In 1985, it was recognised that the Talmud anticipates the modern theory of cooperative games. Each solution corresponds to the nucleolus of an appropriately defined game.

1713

In a letter dated 13 November 1713 James Waldegrave provided the first, known, minimax mixed strategy solution to a two-person game. Waldegrave wrote the letter, about a two-person version of the card game le Her, to Pierre-Remond de Montmort who in turn wrote to Nicolas Bernoulli, including in his letter a discussion of the Waldegrave solution. Waldegrave's solution is a minimax mixed strategy equilibrium, but he made no extension of his result to other games, and expressed concern that a mixed strategy "does not seem to be in the usual rules of play" of games of chance

1838

Publication of Augustin Cournot's Researches into the Mathematical Principles of the Theory of Wealth. In chapter 7, On the Competition of Producers, Cournot discusses the special case of duopoly and utilises a solution concept that is a restricted version of the Nash equilibrium

1871

In the first edition of his book The Descent of Man, and Selection in Relation to Sex Charles Darwin gives the first (implicitly) game theoretic argument in evolutionary biology. Darwin argued that natural section will act to equalize the sex ratio. If, for example, births of females are less common than males, then a newborn female will have better mating prospects than a newborn male and therefore can expect to have more offspring. Thus parents genetically disposed to produce females tend to have more than the average numbers of grandchildren and thus the genes for female-producing tendencies spread, and female births become commoner. As the 1:1 sex ratio is approached, the advantage associated with producing females dies away. The same reasoning holds if males are substituted for females throughout. Therefore 1:1 is the equilibrium ratio.

1881

Publication of Francis Ysidro Edgeworth's Mathematical Psychics: An Essay on the Application of Mathematics to the Moral Sciences. Edgeworth proposed the contract curve as a solution to the problem of determining the outcome of trading between individuals. In a world of two commodities and two types of consumers he demonstrated that the contract curve shrinks to the set of competitive equilibria as the number of consumers of each type becomes infinite. The concept of the core is a generalisation of Edgeworth's contract curve.

1913

The first theorem of game theory asserts that in chess either white can force a win, or black can force a win, or both sides can force at least a draw. This 'theorem' was published by Ernst Zermelo in his paper Uber eine Anwendung der Mengenlehre auf die Theorie des Schachspiels and hence is referred to as Zermelo's Theorem. Zermelo's results were extended and generalised in two papers by Denes Konig and Laszlo Kalmar. The Kalmar paper contains the first proof of Zermelo's theorem since Zermelo's own paper did not give one. An English translation of the Zermelo paper, along with a discussion its significance and its relationship to the work of Konig and Kalmar is contained in Zermelo and the Early History of Game Theory by U. Schwalbe and P. Walker.

1921-27

Emile Borel published four notes on strategic games and an erratum to one of them. Borel gave the first modern formulation of a mixed strategy along with finding the minimax solution for two-person games with three or five possible strategies. Initially he maintained that games with more possible strategies would not have minimax solutions, but by 1927, he considered this an open question as he had been unable to find a counterexample.

1928

John von Neumann proved the minimax theorem in his article Zur Theorie der Gesellschaftsspiele. It states that every two- person zero-sum game with finitely many pure strategies for each player is determined, ie: when mixed strategies are admitted, this variety of game has precisely one individually rational payoff vector. The proof makes involved use of some topology and of functional calculus. This paper also introduced the extensive form of a game.

1930

Publication of F. Zeuthen's book Problems of Monopoly and Economic Warfare. In chapter IV he proposed a solution to the bargaining problem which Harsanyi later showed is equivalent to Nash's bargaining solution.

1934

R.A. Fisher independently discovers Waldegrave's solution to the card game le Her. Fisher reported his work in the paper Randomisation and an Old Enigma of Card Play.

1938

Ville gives the first elementary, but still partially topological, proof of the minimax theorem. Von Neumann and Morgenstern's (1944) proof of the theorem is a revised, and more elementary, version of Ville's proof.

1944

Theory of Games and Economic Behavior by John von Neumann and Oskar Morgenstern is published. As well as expounding two-person zero sum theory this book is the seminal work in areas of game theory such as the notion of a cooperative game, with transferable utility (TU), its coalitional form and its von Neumann-Morgenstern stable sets. It was also the account of axiomatic utility theory given here that led to its wide spread adoption within economics.

1945

Herbert Simon writes the first review of von Neumann-Morgenstern.

1946

The first entirely algebraic proof of the minimax theorem is due to L. H. Loomis's, On a Theorem of von Neumann, paper.

1950

Contributions to the Theory of Games I, H. W. Kuhn and A. W. Tucker eds., published.

1950

In January 1950 Melvin Dresher and Merrill Flood carry out, at the Rand Corporation, the experiment which introduced the game now known as the Prisoner's Dilemma. The famous story associated with this game is due to A. W. Tucker, A Two-Person Dilemma, (memo, Stanford University). Howard Raiffa independently conducted, unpublished, experiments with the Prisoner's Dilemma.

1950

John McDonald's Strategy in Poker, Business and War published. This was the first introduction to game theory for the general reader.

1950-53

In four papers between 1950 and 1953 John Nash made seminal contributions to both non-cooperative game theory and to bargaining theory. In two papers, Equilibrium Points in N- Person Games (1950) and Non-cooperative Games (1951), Nash proved the existence of a strategic equilibrium for non-cooperative games-the Nash equilibrium-and proposed the "Nash program", in which he suggested approaching the study of cooperative games via their reduction to non-cooperative form. In his two papers on bargaining theory, The Bargaining Problem (1950) and Two-Person Cooperative Games (1953), he founded axiomatic bargaining theory, proved the existence of the Nash bargaining solution and provided the first execution of the Nash program.

1951

George W. Brown described and discussed a simple iterative method for approximating solutions of discrete zero-sum games in his paper Iterative Solutions of Games by Fictitious Play.

1952

The first textbook on game theory was John Charles C. McKinsey, Introduction to the Theory of Games.

1952

Merrill Flood's report, (Rand Corporation research memorandum, Some Experimental Games, RM-789, June), on the 1950 Dresher/Flood experiments appears.

1952

The Ford Foundation and the University of Michigan sponsor a seminar on the "Design of Experiments in Decision Processes" in Santa Monica. This was the first experimental economics/experimental game theory conference

1952-53

The notion of the Core as a general solution concept was developed by L. S. Shapley (Rand Corporation research memorandum, Notes on the N-Person Game III: Some Variants of the von-Neumann-Morgenstern Definition of Solution, RM- 817, 1952) and D. B. Gillies (Some Theorems on N-Person Games, Ph.D. thesis, Department of

Mathematics, Princeton University, 1953). The core is the set of allocations that cannot be improved upon by any coalition.

1953

Lloyd Shapley in his paper A Value for N-Person Games characterised, by a set of axioms, a solution concept that associates with each coalitional game,v, a unique out-come, v. This solution in now known as the Shapley Value.

1953

Lloyd Shapley's paper Stochastic Games showed that for the strictly competitive case, with future payoff discounted at a fixed rate, such games are determined and that they have optimal strategies that depend only on the game being played, not on the history or even on the date, ie: the strategies are stationary.

1953

Extensive form games allow the modeller to specify the exact order in which players have to make their decisions and to formulate the assumptions about the information possessed by the players in all stages of the game. H. W. Kuhn's paper, Extensive Games and the Problem of Information includes the formulation of extensive form games which is currently used, and also some basic theorems pertaining to this class of games.

1953

Contributions to the Theory of Games II, H. W. Kuhn and A. W. Tucker eds., published.

1954

One of the earliest applications of game theory to political science is L. S. Shapley and M. Shubik with their paper A Method for Evaluating the Distribution of Power in a Committee System. They use the Shapley value to determine the power of the members of the UN Security Council.

1954-55

Differential Games were developed by Rufus Isaacs in the early 1950s. They grew out of the problem of forming and solving military pursuit games. The first publications in the area were Rand Corporation research memoranda, by Isaacs, RM-1391 (30 November 1954), RM-1399 (30 November 1954), RM-1411 (21 December 1954) and RM-1486 (25 March 1955) all entitled, in part, Differential Games.

1955

One of the first applications of game theory to philosophy is R. B. Braithwaite's Theory of Games as a Tool for the Moral Philosopher.

1957

Games and Decisions: Introduction and Critical Survey by Robert Duncan Luce and Howard Raiffa published.

1957

Contributions to the Theory of Games III, M. A. Dresher, A. W. Tucker and P. Wolfe eds., published.

1959

The notion of a Strong Equilibrium was introduced by R. J. Aumann in the paper Acceptable Points in General Cooperative N-Person Games.

1959

The relationship between Edgeworth's idea of the contract curve and the core was pointed out by Martin Shubik in his paper Edgeworth Market Games. One limitation with this paper is that Shubik worked within the confines of TU games whereas Edgeworth's idea is more appropriately modelled as an NTU game.

1959

Contributions to the Theory of Games IV, A. W. Tucker and R. D. Luce eds., published.

1959

Publication of Martin Shubik's Strategy and Market Structure: Competition, Oligopoly, and the Theory of Games. This was one of the first books to take an explicitly non-cooperative game theoretic approach to modelling oligopoly. It also contains an early statement of the Folk Theorem.

Late 50's

Near the end of this decade came the first studies of repeated games. The main result to appear at this time was the Folk Theorem. This states that the equilibrium outcomes in an infinitely repeated game coincide with the feasible and strongly individually rational outcomes of the one-shot game on which it is based. Authorship of the theorem is obscure.

1960

The development of NTU (non-transferable utility) games made cooperative game theory more widely applicable. Von Neumann and Morgenstern stable sets were investigated in the NTU context in the Aumann and Peleg paper Von Neumann and Morgenstern Solutions to Cooperative Games Without Side Payments.

1960

Publication of Thomas C. Schelling's The Strategy of Conflict. It is in this book that Schelling introduced the idea of a focal-point effect.

1961

The first explicit application to evolutionary biology was by R. C. Lewontin in Evolution and the Theory of Games.

1961

The Core was extended to NTU games by R. J. Aumann in his paper The Core of a Cooperative Game Without Side Payments.

1962

In their paper College Admissions and the Stability of Marriage, D. Gale and L. Shapley asked whether it is possible to match m women with m men so that there is no pair consisting of a woman and a man who prefer each other to the partners with whom they are currently matched. Game theoretically the question is, does the appropriately defined NTU coalitional game have a non-empty core? Gale and Shapley proved not only non-emptiness but also provided an algorithm for finding a point in it.

1962

One of the first applications of game theory to cost allocation is Martin Shubik's paper Incentives, Decentralized Control, the Assignment of Joint Costs and Internal Pricing. In this paper Shubik argued that the Shapley value could be used to provide a means of devising incentive-compatible cost assignments and internal pricing in a firm with decentralised decision making.

1962

An early use of game theory in insurance is Karl Borch's paper Application of Game Theory to Some Problems in Automobile Insurance. The article indicates how game theory can be applied to determine premiums for different classes of insurance, when required total premium for all classes is given. Borch suggests that the Shapley value will give reasonable premiums for all classes of risk.

1963

O. N. Bondareva established that for a TU game its core is non-empty iff it is balanced. The reference, which is in Russian, translates as Some Applications of Linear Programming Methods to the Theory of Cooperative Games.

1963

In their paper A Limit Theorem on the Core of an Economy G. Debreu and H. Scarf generalised Edgeworth, in the context of a NTU game, by allowing an arbitrary number of commodities and an arbitrary but finite number of types of traders.

1964

Robert J. Aumann further extended Edgeworth by assuming that the agents constitute a (non-atomic) continuum in his paper Markets with a Continuum of Traders.

1964

The idea of the Bargaining Set was introduced and discussed in the paper by R. J. Aumann and M. Maschler, The Bargaining Set for Cooperative Games. The bargaining set includes the core but unlike it, is never empty for TU games.

1964

Carlton E. Lemke and J.T. Howson, Jr., describe an algorithm for finding a Nash equilibrium in a bimatrix game, thereby giving a constructive proof of the existence of an equilibrium point, in their paper Equilibrium Points in Bimatrix Games. The paper also shows that, except for degenerate situations, the number of equilibria in a bimatrix game is odd.

1965

Publication of Rufus Isaacs's Differential Games: A Mathematical Theory with Applications to Warfare and Pursuit, Control and Optimization.

1965

R. Selten, Spieltheoretische Behandlung eines Oligopolmodells mit Nachfragetraegheit. In this article Selten introduced the idea of refinements of the Nash equilibrium with the concept of (subgame) perfect equilibria.

1965

The concept of the Kernel is due to M. Davis and M. Maschler, The Kernel of a Cooperative Game. The kernel is always included in the bargaining set but is often much smaller.

1966

Infinitely repeated games with incomplete information were born in a paper by R. J. Aumann and M. Maschler, Game-Theoretic Aspects of Gradual Disarmament.

1966

In his paper A General Theory of Rational Behavior in Game Situations John Harsanyi gave the, now, most commonly used definition to distinguish between cooperative and non-cooperative games. A game is cooperative if commitments--agreements, promises, threats--are fully binding and enforceable. It is non-cooperative if commitments are not enforceable.

1967

Lloyd Shapley, independently of O.N. Bondareva, showed that the core of a TU game is non-empty iff it is balanced in his paper On Balanced Sets and Cores.

1967

In the articleThe Core of a N-Person Game, H. E. Scarf extended the notion of balancedness to NTU games, then showed that every balanced NTU game has a non-empty core.

1967-68

In a series of three papers, Games with Incomplete Information Played by 'Bayesian' Players, Parts I, II and III, John Harsanyi constructed the theory of games of incomplete information. This laid the theoretical groundwork for information economics that has become one of the major themes of economics and game theory.

1968

The long-standing question as to whether stable sets always exist was answered in the negative by William Lucas in his paper A Game with no Solution.

1969

David Schmeidler introduced the Nucleolus in this paper The Nucleolus of a Characteristic Game. The Nucleolus always exists, is unique, is a member of the Kernel and for any non- empty core is always in it.

1969

Shapley defined a value for NTU games in his article Utility Comparison and the Theory of Games.

1969

For a coalitional game to be a market game it is necessary that it and all its subgames have non-empty cores, ie: that the game be totally balanced. In Market Games L. S. Shapley and Martin Shubik prove that this necessary condition is also sufficient.

1972

International Journal of Game Theory was founded by Oskar Morgenstern.

1972

The concept of an Evolutionarily Stable Strategy (ESS), was introduced to evolutionary game theory by John Maynard Smith in an essay Game Theory and The Evolution of Fighting. The ESS concept has since found increasing use within the economics (and biology!) literature.

1973

In the traditional view of strategy randomization, the players use a randomising device to decide on their actions. John Harsanyi was the first to break away from this view with his paper Games with Randomly Disturbed Payoffs: A New Rationale for Mixed Strategy Equilibrium Points. For Harsanyi nobody really randomises. The appearance of randomisation is due to the payoffs not being exactly known to all; each player, who knows his own payoff exactly, has a unique optimal action against his estimate of what the others will do.

1973

The major impetus for the use of the ESS concept was the publication of J. Maynard Smith and G. Price's paper The Logic of Animal Conflict.

1973

The revelation principle can be traced back to Gibbard's paper Manipulation of Voting Schemes: A General Result

1974

Publication of R. J. Aumann and L. S. Shapley's book Values of Non-Atomic Games. It deals with values for large games in which all the players are individually insignificant (non-atomic games).

1974

R. J. Aumann proposed the concept of a correlated equilibrium in his paper Subjectivity and Correlation in Randomized Strategies.

1975

The introduction of trembling hand perfect equilibria occurred in the paper Reexamination of the Perfectness Concept for Equilibrium Points in Extensive Games by Reinhard Selten. This paper was the true catalyst for the 'refinement industry' that has developed around the Nash equilibrium.

1975

E. Kalai and M. Smorodinsky, in their article Other Solutions to Nash's Bargaining Problem, replace Nash's independence of irrelevant alternatives axiom with a monotonicity axiom. The resulting solution is known as the Kalai-Smorodinsky solution.

1975

In his paper Cross-Subsidization: Pricing in Public Enterprises, G. Faulhaber shows that the set of subsidy-free prices are those prices for which the resulting revenue (ri = piqi for given demand levels qi) vector lies in the core of the cost allocation game.

1976

An event is common knowledge among a set of agents if all know it and all know that they all know it and so on ad infinitum. Although the idea first appeared in the work of the philosopher D. K. Lewis in the late 1960s it was not until its formalisation in Robert Aumann's Agreeing to Disagree that game theorists and economists came to fully appreciate its importance.

1977

S. C. Littlechild and G. F. Thompson are among the first to apply the nucleolus to the problem of cost allocation with their article Aircraft Landing Fees: A Game Theory Approach. They use the nucleolus, along with the core and Shapley value, to calculate fair and efficient landing and take-off fees for Birmingham airport.

1981

Elon Kohlberg introduced the idea of forward induction in a conference paper Some Problems with the Concept of Perfect Equilibria.

1981

R. J. Aumann published a Survey of Repeated Games. This survey firstly proposed the idea of applying the notion of an automaton to describe a player in a repeated game. A second idea from the survey is to study the interactive behaviour of bounded players by studying a game with appropriately restricted set of strategies. These ideas have given birth to a large and growing literature.

1982

David M. Kreps and Robert Wilson extend the idea of a subgame perfect equilibrium to subgames in the extensive form that begin at information sets with imperfect information. They call this extended idea of equilibrium sequential. It is detailed in their paper Sequential Equilibria.

1982

A. Rubinstein considered a non-cooperative approach to bargaining in his paper Perfect Equilibrium in a Bargaining Model. He considered an alternating-offer game were offers are made sequentially until one is accepted. There is no bound on the number of offers that can be made but there is a cost to delay for each player. Rubinstein showed that the subgame perfect equilibrium is unique when each player's cost of time is given by some discount factor delta.

1982

Publication of Evolution and the Theory of Games by John Maynard Smith.

1984

Following the work of Gale and Shapley, A. E. Roth applied the core to the problem of the assignment of interns to hospitals. In his paper The Evolution of the Labour Market for Medical Interns and Residents: A Case Study in Game Theory he found that American hospitals developed in 1950 a method of assignment that is a point in the core.

1984

The idea of a rationalizability was introduced in two papers; B. D. Bernheim, Rationalizable Strategic Behavior and D. G. Pearce, Rationalizable Strategic Behavior and the Problem of Perfection.

1984

Publication of The Evolution of Cooperation by Robert Axelrod.

1985

For a Bayesian game the question arises as to whether or not it is possible to construct a situation for which there is no sets of types large enough to contain all the private information that players are supposed to have. In their paper, Formulation of Bayesian Analysis for Games with Incomplete Information, J.-F. Mertens and S. Zamir show that it is not possible to do so.

1985-86

Following Aumann, the theory of automata is now being used to formulate the idea of bounded rationality in repeated games. Two of the first articles to take this approach were A. Neyman's 1985 paper Bounded Complexity Justifies Cooperation in the Finitely Repeated Prisoner's Dilemma and A. Rubinstein's 1986 article Finite Automata Play the Repeated Prisoner's Dilemma.

1986

In their paper On the Strategic Stability of Equilibria Elon Kohlberg and Jean-Francois Mertens deal with the problem of he refinement of Nash equilibria in the normal form, rather than the extensive form of a game as with the Selten and Kreps and Wilson papers. This paper is also one of the first, published, discussions of the idea of forward induction.

1988

John C. Harsanyi and Reinhard Selten produced the first general theory of selecting between equilibria in their book A General Theory of Equilibrium Selection in Games. They provide criteria for selecting one particular equilibrium point for any non-cooperative or cooperative game.

1988

With their paper The Bayesian Foundations of Solution Concepts of Games Tan and Werlang are among the first to formally discuss the assumptions about a player's knowledge that lie behind the concepts of Nash equilibria and rationalizability.

1988

One interpretation of the Nash equilibrium is to think of it as an accepted (learned) 'standard of behaviour' which governs the interaction of various agents in repetitions of similar situations. The problem then arises of how agents learn the equilibrium. One of the earliest works to attack the learning problem was Drew Fudenberg and David Kreps's A Theory of Learning, Experimentation and Equilibria, (MIT and Stanford Graduate School of Business, unpublished), which uses an learning process similar to Brown's fictitious play, except that player occasionally experiment by choosing strategies at random, in the context of iterated extensive form games. Evolutionary game models are also commonly utilised within the learning literature.

1989

The journal Games and Economic Behavior founded.

1990

The first graduate level microeconomics textbook to fully integrate game theory into the standard microeconomic material was David M. Krep's A Course in Microeconomic Theory.

1990

In the article Equilibrium without Independence Vincent Crawford discusses mixed strategy Nash equilibrium when the players preferences do not satisfy the assumptions necessary to be represented by expected utility functions.

1991

An early published discussion of the idea of a Perfect Bayesian Equilibrium is the paper by D. Fudenberg and J. Tirole, Perfect Bayesian Equilibrium and Sequential Equilibrium.

1992

Publication of the Handbook of Game Theory with Economic Applications, Volume 1 edited by Robert J. Aumann and Sergiu Hart.

1994

Game Theory and the Law by Douglas G. Baird, Robert H. Gertner and Randal C. Picker is one of the first books in law and economics to take an explicitly game theoretic approach to the subject.

1994

Publication of the Handbook of Game Theory with Economic Applications, Volume 2 edited by Robert J. Aumann and Sergiu Hart.

1994

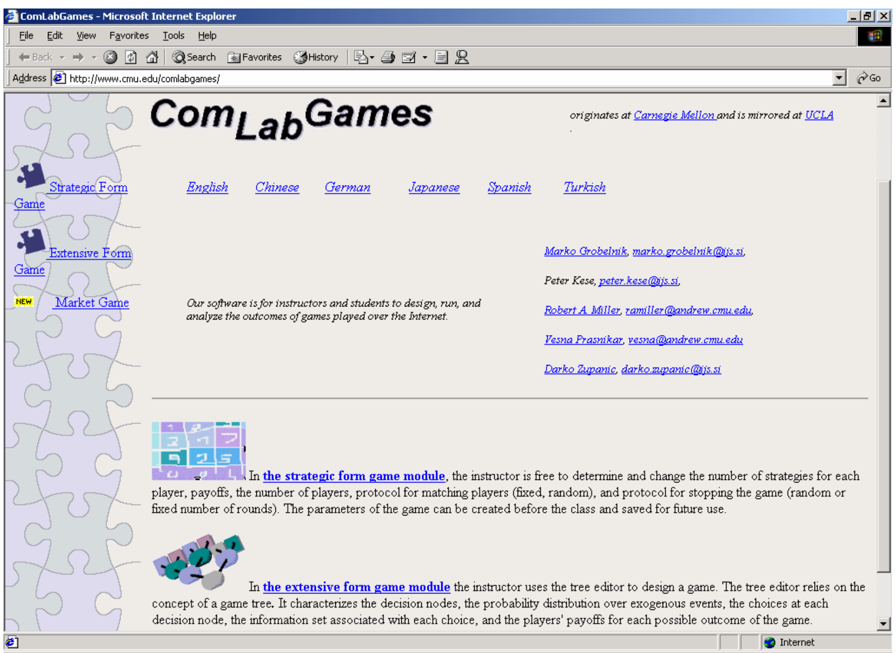
The Sveriges Riksbank (Bank of Sweden) Prize in Economic Sciences in Memory of Alfred Nobel was award to John Nash, John C. Harsanyi and Reinhard Selten for their contributions to Game Theory.

Appendix B – ComLabGames and Experimental Methods

The site **ComLabGames** (http://www.cmu.edu/comlabgames/), created by prof. Robert Miller and Vesna Prasnikar, provides important software for instructors and students to design, run, and analyze the outcomes of games played over the Internet.

In **the strategic form game module**, the instructor is free to determine and change the number of strategies for each player, payoffs, the number of players, protocol for matching players (fixed, random), and protocol for stopping the game (random or fixed number of rounds). The parameters of the game can be created before the class and saved for future use.

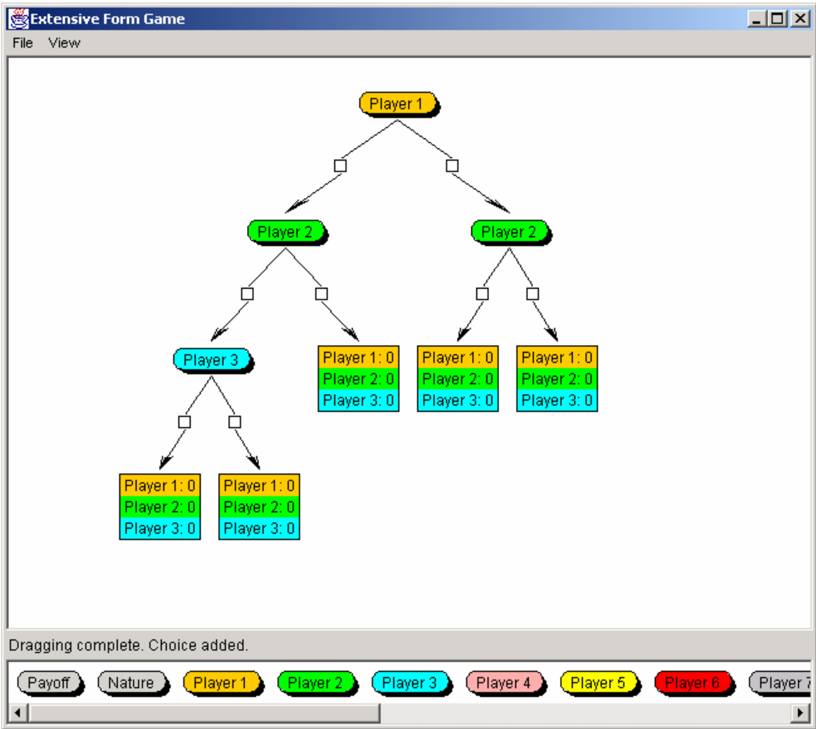
In **the extensive form game module** the instructor uses the tree editor to design a game. The tree editor relies on the concept of a game tree. It characterizes the decision nodes, the probability distribution over exogenous events, the choices at each decision node, the information set associated with each choice, and the players' payoffs for each possible outcome of the game.

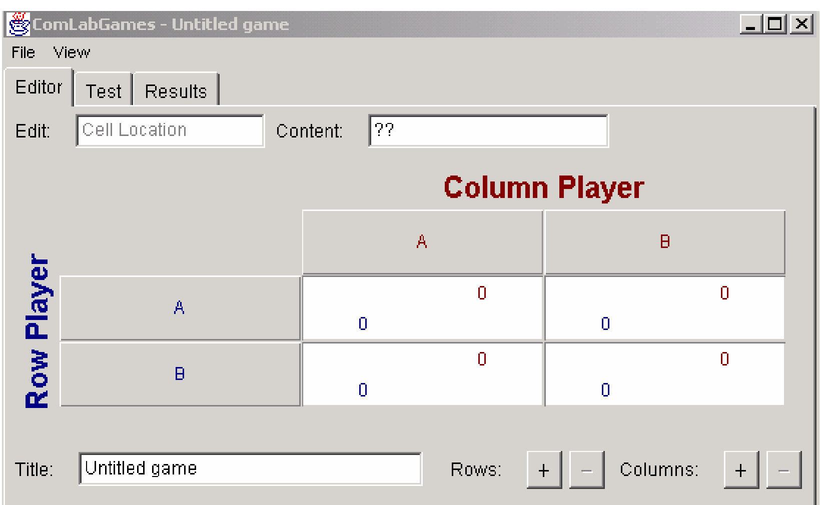


Hosted at Carnegie Mellon, with mirrors in other places, the extensive and strategic modules are used in the class 45-922 Experimental Methods for Business Strategy, offered by prof. Miller and Prasnikar.

As a free tool in the internet, they can be used by other schools in different languages.

Below are the tools in the designing mode, where students can model and test their games, including players, actions, payoffs, probabilities, etc. The game can be played online by multiple users. The software also keeps a log of all moves to further analysis and statistics of behaviors.





Besides schools, companies also can use those tools to train their managers.